BOOK PROJECT, INFORMALLY ENTITLED "LOOP, DE-LOOP"

This book would be aimed at filling a need in the 2nd/3rd-year (transition) graduate topology curriculum, in part inspired by Fulton-Harris's Representation Theory book. It would be based on a course taught at Brown and, on an individual basis, at the University of Oregon. The unifying theme is the study of mapping spaces and classifying spaces through models (mainly simplicial and configuration space). Many other topics would be weaved in out of necessity. Concrete examples are to be emphasized. This book could also serve as an introduction to working mathematicians from other fields (who may be interested in string topology, loop groups, spectra, or other topics and desire an elementary treatment). It is meat to be more elementary than Kallel's book, and a great expansion of Carlsson-Milgram survey.

(1) One-fold loop spaces ($\Omega^n X$ for $n = 1$).
   (a) Historical methods - Morse theory and Leray-Serre.
   (b) James' theorem on $\Omega\Sigma X$.
   (c) The geometric Adams-Hilton model and the Eilenberg-Moore spectral sequence.

(2) Classifying spaces ($\Omega^n X$ for $n = -1$, so to speak).
   (a) Classical motivation - principal G-bundles and vector bundles.
   (b) Classical models - Stiefel frames and Milnor's join construction.
   (c) The simplicial model and the bar construction.
   (d) An invitation to group cohomology.

(3) Infinite loop spaces (The existence of $\Omega^n X$ for $n = -\infty$, so to speak).
   (a) The relationship between infinite loop spaces, cohomology theories, and spectra.
   (b) Simplicial construction of Eilenberg-MacClane spectra.
   (c) Survey of other examples of spectra.

(4) Cosimplicial models for mapping spaces. (Can be used to study $\Omega^n X$ for any positive n).
   (a) The cosimplicial spectral sequence for homotopy groups of a mapping space - a tautology.
   (b) The free module functor and homology (the Dold-Thom theorem).
   (c) The generalized Eilenberg-Moore spectral sequence.
   (d) Some examples.

(5) Operads and recognition principles (When does $X \simeq \Omega^n Y$ for some Y?).
   (a) Homotopy monoids and Stasheff's Thesis.
   (b) Operad actions.
   (c) $\Omega^n \Sigma^n X$ as a free algebra over the little disks.
   (d) The Barratt-Priddy-Quillen-Segal theorem.
   (e) The recognition theorems.

(6) Homology of iterated loop spaces (Combining techniques to understand $H_*(\Omega^n X)$).
   (a) Homology of configuration spaces and Poisson algebras.
   (b) Dyer-Lashof and Steenrod operations.
   (c) Connecting with cosimplicial methods, through McClure-Smith's work.

   (d) Some examples.

(7) Appendices
   (a) Spectral sequences
      (i) Of a bicomplex.
      (ii) Of a filtration.
      (iii) Example: Leray-Serre.

   (b) Simplicial objects and homotopy limts and colimits
      (i) Simplicial sets and realization.
      (ii) Simplicial spaces.
      (iii) Cosimplicial spaces, totalization, and mapping spaces.
      (iv) (Co)simplicial objects in general.
      (v) Classifying spaces of categories.
      (vi) Homotopy limits and colimits.