Math 457; Due April 2, 2010: Getting to know Mathematica

Mathematica is available in various ways at the U of O. The easiest way to use it is probably to do so at one of the computer labs on campus—for instance at the Science Library ITC. The computers in these labs are most likely set up so that you can just click on an icon, but you can probably also type "mathematica" at a unix prompt. I think Mathematica is also available on the computers in Hilbert Space (on the first floor of Deady).

Note that in the windows version of Mathematica commands will not be executed until you type <Shift-Return> (rather than just <Return> as you might naively expect).

1. Mathematica can perform calculations. Below are some samples. We write E instead of e, Pi instead of π, Log[x] instead of $\ln x$, Sin[x] instead of $\sin x$, and Sqrt[x] instead of $\sqrt{x}$. Notice that computed values can be named and later used.

$$
\begin{aligned}
\mathrm{In}[1] &:= \mathbf{E\hat{}Log[5]} \\
\mathrm{Out}[1] &:= 5 \\
\mathrm{In}[9] &:= \mathbf{A1{=}Sqrt[2{+}Sin[5]]} \\
\mathrm{Out}[9] &:= \sqrt{2+\mathrm{Sin}[5]} \\
\mathrm{In}[10] &:= \mathbf{N[A1]} \\
\mathrm{Out}[10] &:= 1.02033 \\
\mathrm{In}[14] &:= \mathbf{Sqrt[2.0{+}Sin[5]]} \\
\mathrm{Out}[14] &:= 1.02033
\end{aligned}
$$

   Mathematica uses square brackets for arguments of functions, e.g. Sqrt[5], and round brackets for grouping, e.g. $(2+3)^5$. Try typing the two commands

   (1+3)/9
   N[(1+3)/9]

   What do you think the function $N[x]$ does? Try typing $N[\mathrm{Pi}, 100]$.

   Compute the following. Where possible, find exact values and decimal approximations.

$$
2+5, \qquad \frac{2+7}{3+3}, \qquad \left(2+\frac{3}{5}\right)^3, \qquad \frac{2+\sin(\pi/2)}{\sqrt{9}}, \qquad 100!, \qquad \sqrt{\ln(\ln(17))}, \qquad \left(\frac{13}{54}\right)^{50}
$$

2. You can define functions and later use them; an example is shown below. In the definition, notice that there is an underscore character following x on the left, but not on the right.

A common beginner's mistake (which I still make) is to forget to use the underscore when defining functions. Consider yourself warned!

In[5] := **f[x_] := Sqrt[x+1]**
In[6] := **f[3]**
Out[6] := 2
In[7] := **f[x]^2**
Out[7] := $1 + x$

Define $g(x) = \sin(x^2)$. Then compute $g(0), g(\sqrt{\pi/2})$, and $g(1.72)$. Find a decimal approximation for $g(2)$.

3. The expression

$$\sum_{i=1}^{n} f(i)$$

is written Sum[f[i], {i, 1, n}] in Mathematica. For example, Sum[i^2, {i, 2, 4}] computes $2^2 + 3^2 + 4^2 = 29$. Find the following sums:

$$\sum_{k=0}^{5} 2^k \qquad \sum_{i=1}^{3} \frac{1}{i} \qquad \sum_{i=1}^{100} \frac{1}{i} \qquad \sum_{s=1}^{100} \frac{1}{s^2}$$

Compute a decimal approximation for the quotient

$$\frac{\pi^2}{\sum_{s=1}^{n} \frac{1}{s^2}}$$

when $n = 100$, $n = 1000$, $n = 5000$. The sequence converges as n gets very large, but you'll notice that it converges pretty slowly.

Given the computations you just made, make an educated guess for the value of $\sum_{s=1}^{\infty} \frac{1}{s^2}$. Euler found this value (and proved that it was correct) long before computers were around—all his experimental evidence for this fact was obtained by hand calculations.

4. To use Mathematica to iterate a function, you can enter the following small program:

```
Iterate[a_ , n_] := Module[ {x,i},
                    x=a;
```

```
For[i=1, i<n, i=i+1,
x=f[x];
Print[x];
];
];
```

To use the above program you must first have defined a function $f[x]$. Then Iterate[a,n] computes $f(a), f(f(a)), \ldots, f^{n-1}(a)$. For instance, try the following commands:

f[x_] := $x^2 - 1.2$
Iterate[0,100]

Play around with what happens when you use different values of a and n.

In fact, Mathematica has a version of the Iterate program built into it. Try typing

NestList[f,0,100].

WARNING: Mathematica remembers previous calculations. If you set $x = 3$ and later graph $f(x)$, you'll get a horizontal line of height $f(3)$. This can be confusing. The command Clear[x] causes Mathematica to forget the old value of x. Several values can be cleared at once, as in Clear[x, y, this, that, C1, D3]. Remember this!

5. We'll often use Mathematica to graph functions. The basic syntax is Plot[f[x], {x, a, b}]. For example, the graph on the left is a plot of $\sin(x)$ from 0 to 10 and the plot on the right is a graph of $x^2 + 1$ from -1 to 1.

Examine the picture on the right. Surprising? Mathematica has three habits which are bad for us (it actually has many more than three—I just mean three in the present situation). It uses different scales on the two axes, it chooses a reasonable portion of the xy-plane to show, and it draws the horizontal axis at a crazy spot if $y = 0$ is not in the picture.

We can solve these problems by adding extra words to the plot command. The command PlotRange $\rightarrow$ {a, b} tells Mathematica to graph using $a \leq y \leq b$ and the command AspectRatio $\rightarrow$ Automatic tells Mathematica to use the same scale on both axes. Below is a picture of $x^2 + 1$ generated by using the command

```
Plot[x^2 + 1, {x, -1, 1}, PlotRange->{0, 2}, AspectRatio->Automatic]
```

Please choose several functions and plot them to see how this works. Try $\sqrt{x}$ from 0 to 1 and $\sqrt{x}$ from -1 to 1. Try $\ln(x)$ from -1 to 1.

6. Several functions can be plotted on the same graph by making a list of the functions. This will be very useful for our course. Below is a plot of x and $x^3 - x^2 - x$ on the same graph constructed with the command

   ```
   Plot[ {x, x^3 - x^2 - x}, {x, -3, 3}, PlotRange->{-3, 3},
             AspectRatio->Automatic]
   ```

   Suppose we want to study the dynamics of $f(x) = x^3 - x^2 - x$. What are the fixed points of f?

7. Let $f(x) = x^2 - 1$. Plot x and $f(x)$ on the same graph. Explain the significance of the points where these graphs cross. Plot x and $f(f(x))$ on the same graph. Explain the significance (in our course) of the points where these graphs cross. Plot $x, f(x)$, and $f(f(x))$ on the same graph.

8. Consider the function

$$f(x) = \begin{cases} 2x & \text{if } x < \frac{1}{2} \\ \\ 2x - 1 & \text{if } \frac{1}{2} \le x < 1 \end{cases}$$

This function can be defined and graphed in the manner shown below. Here Which[test, value, test, value, test, value] is a Mathematica function which applies the first test to x, yields the first value if x passes, otherwise applies the second test to x, yields the second value if x passes, otherwise ...

```
f[x_] := Which[x < 1/2, 2 x, True, 2 x - 1];
Plot[{x, f[x]}, {x, 0, 1}, PlotRange->{0, 1}, AspectRatio->Automatic]
```

Please try this. Mathematica has the bad habit of making a vertical line where functions have jumps, but we can live with it.

9. We can use Mathematica to plot the orbit diagram for the function $f(x) = x^2 + c$, as c ranges over a given interval. Enter the following program:

```
OrbDiag[n_] := Module[ {mylist, c, x, i},
               mylist = {};
               For[c = -2.0, c <= .25, c = c + .05,
               x = 0;
               For[i = 1, i < 100, i = i + 1,
                          x = x^2 + c];
               For[i = 1, i < n, i = i + 1,
                          x = x^2 + c;
               mylist = Append[mylist, {c, x}];
               ];
               ];
               ListPlot[mylist]
               ];
```

For values of c between -2 and 0.25, this program computes the first 100 iterates of 0 under the functions $f(x) = x^2 + c$, throws them away, and then makes a list containing the next n iterates. The points in this list are plotted at the end of the program. Enter the command

OrbDiag[50]

to see it work.

If you are brave, you can zoom in on parts of this diagram by playing with the limits of c in the first "For" loop. We will do this in a couple of weeks.