

Math 647 Final

Answer as many questions as you can! Make sure you state clearly any theorems from class that you use.

Part I. Definitions.

1. What is a *solvable* group?

Part II. True or False. Justify your answers briefly.

1. The subgroup B of $GL_2(\mathbb{F}_3)$ consisting of all invertible upper triangular matrices is isomorphic to the alternating group A_4 .
2. Suppose that $N_1 \trianglelefteq G_1$ and $N_2 \trianglelefteq G_2$. If $G_1 \cong G_2$ and $N_1 \cong N_2$ then $G_1/N_1 \cong G_2/N_2$.
3. All subgroups of the quaternion group Q_8 of order 8 are normal.
4. Let F be the free group on the set $X = \{a, b\}$. Let N be the normal subgroup of F generated by the element ab . Then, $F/N \cong \mathbb{Z}$.
5. If G is a primitive permutation group on a finite set X , then the action of G on X is 2-transitive.

Part III. Longer problems.

1. Use the Sylow theorems to show that there is no simple group of order 144.
2. Let p be an odd prime. Classify all finite groups of order $2p$ up to isomorphism.
3. If N is a non-trivial normal subgroup of a finite nilpotent group G , prove that $N \cap Z(G) \neq \{1\}$.