

# Contents

<b>0</b>	<b>Categories</b>	<b>1</b>
0.1	Categories . . . . .	1
0.2	Examples of categories . . . . .	2
0.3	Products and coproducts . . . . .	3
0.4	Functors and natural transformations . . . . .	5
0.5	*Additive categories* . . . . .	7
<b>1</b>	<b>Groups</b>	<b>11</b>
1.1	The symmetric group . . . . .	11
1.2	Isomorphism theorems . . . . .	15
1.3	Group actions . . . . .	18
1.4	Finite subgroups of $SO(3)$ . . . . .	20
1.5	The Sylow theorems . . . . .	23
1.6	Semidirect products . . . . .	24
1.7	Projective unimodular groups . . . . .	27
1.8	*Jordan-Hölder theorem* . . . . .	30
1.9	Solvable and nilpotent groups . . . . .	32
1.10	Free groups . . . . .	35
<b>2</b>	<b>Rings</b>	<b>41</b>
2.1	Homomorphisms and ideals . . . . .	41
2.2	Maximal and prime ideals . . . . .	43
2.3	Euclidean domains and PIDs . . . . .	45
2.4	Unique factorization domains . . . . .	48
2.5	Localization of rings . . . . .	49
2.6	Factorization in polynomial rings . . . . .	50
<b>3</b>	<b>Modules</b>	<b>53</b>
3.1	Modules, submodules and homomorphisms . . . . .	53
3.2	Direct products and direct sums . . . . .	56
3.3	Short exact sequences . . . . .	58
3.4	Semisimple modules . . . . .	58
3.5	Free modules . . . . .	60
3.6	The Krull-Schmidt theorem . . . . .	62
3.7	Finitely generated modules over PIDs . . . . .	66
3.8	Applications to linear algebra . . . . .	72
3.9	Tensor products . . . . .	78
3.10	Projectives and injectives . . . . .	83
3.11	Adjoint functors . . . . .	86
3.12	Morita equivalence . . . . .	92

<b>4</b>	<b>Algebras</b>	<b>97</b>
4.1	Definition . . . . .	97
4.2	Some multilinear algebra . . . . .	99
4.3	Chain conditions . . . . .	104
4.4	Wedderburn structure theorems . . . . .	106
4.5	The Jacobson radical . . . . .	110
4.6	Character theory of finite groups . . . . .	113
4.7	Burnside's $p^a q^b$ theorem . . . . .	123
<b>5</b>	<b>Fields</b>	<b>129</b>
5.1	Field extensions . . . . .	129
5.2	Transcendental extensions . . . . .	131
5.3	Splitting fields . . . . .	133
5.4	*Normal extensions* . . . . .	135
5.5	*Separable extensions* . . . . .	136
5.6	*The fundamental theorem of Galois theory* . . . . .	138
5.7	*Radical extensions* . . . . .	140
5.8	*Finite fields* . . . . .	144
<b>6</b>	<b>Commutative algebra</b>	<b>147</b>
6.1	Ring extensions . . . . .	147
6.2	Hilbert's basis theorem . . . . .	150
6.3	Nullstellensatz . . . . .	152
6.4	Affine varieties . . . . .	155
6.5	The Zariski topology . . . . .	157
6.6	Irreducible components and dimension . . . . .	158