

## Math 392: Homework 5 (due W February 20)

**Part I: reading.** Read section 9.3. In this section you'll find the proof due to Gauss that there are no integer solutions to the equation  $x^3 + y^3 = z^3$ . This is the  $n = 3$  case of "Fermat's last theorem". I thought about trying to go over this  $n = 3$  proof in class but decided it was too long and technical, so don't expect to find it easy to follow in the book either! But see how all the things like EDs, PIDs and UFDs are being used in the proof in essential ways.

**Part II: problems.** I've decided to set a different sort of homework. What I'd like from you is a complete, comprehensive, and beautifully written solution to the following problems.

1. Describe how to calculate all integer solutions to the Diophantine equation  $2x^2 - y^2 = 1$  and prove that you have them all.
2. Describe how to calculate all integer solutions to the Diophantine equation  $x^2 + y^2 = z^2$  (better known as "Pythagorean triples") and prove that you have them all.

I would expect that if done properly each answer would take at least one full page of nicely written text!