392 Homework 8

DUE ON FRIDAY OF NEXT WEEK – NOT THE USUAL WEDNESDAY!!!

- The last topic this term is to do with the material in section 4.3 in the book. So we are going to take another look at the Gaussian integers – which we studied quite a lot last term – and show you some beautiful applications of the Gaussian integers and the ring theory we have developed. You should aim to read section 4.3 in the book at this point ...
- Note by the end of term we will have covered all of the first four chapters in the text book. It would be a good idea to begin reviewing everything so far (!!!!) in preparation for the final exam this term. I know you probably don't have time – but it is so important to try every now and then to take a look at the subject as a whole to see how everything fits together. Only now are you really in a good position to understand the point ...
- By the way this is the last homework that will be graded this term. In a while, I will give out a review sheet for the final though it will not be graded.
- Section 4.3: 1(a)(b), 2(a)(b), 5, 8, 10(a)(b) (When you've done question 10, you should know which out of the Gaussian integers $1+i$ and $3+i$ is irreducible in $\mathbb{Z}[i]$ ...).
- In this section we have proved that an odd prime can be written as the sum of two squares if and only if $p \equiv 1 \pmod 4$. It is natural then to ask which *natural numbers* (not just primes) can be written as a sum of two squares. You might like to try to come up with a conjecture for yourself about this, but don't try too long – read Exercise 15.
- The number 7 cannot be written as a sum of two or of three squares, but it can be written as $2^2+1^2+1^2+1^2$ as a sum of four squares. In fact, there is a famous theorem that shows that *any* positive integer can be expressed as a sum of four squares, i.e.

  $$n = a^2 + b^2 + c^2 + d^2$$

  for integers $a, b, c, d \geq 0$. Convince yourself that this is at least plausible by checking that you can write each of the numbers $1, \dots, 20$ as a sum of four squares. Which ones can you write as a sum of just three squares? Any conjectures?