

392 HOMEWORK 7

- For the week and a half we are going to prove some of the patterns you've noticed in factoring $x^n - 1$. Then in the last week and a half I want to cover the material in section 4.3 in the book. Hopefully both these topics will help you get a better understanding of the fundamental ideas we have been discussing in fairly concrete situations.
- You should REREAD sections 4.1 and 4.2 in the book. You should have read these before already – but now is a good moment to look back at them – it should start to make some sense this time!
- Exercises 4.1: 6, 18(a)(b)(c)
- Exercises 4.2: 3(e)
- Exercises 3.3: 3(b)(d), 8, 10.
- Compute the cyclotomic polynomials $\Phi_n(x)$ for $n = 1, 2, \dots, 16$ (you should be able to read the first 12 off from the table of factors of $(x^n - 1)$ you created last week – remember $\Phi_n(x)$ is one of its irreducible factors of degree $\phi(n)$.) Question: for which n are ALL the coefficients of the polynomial $\Phi_n(x)$ positive? Do you see any pattern? Probably not, since as far as I am aware it is an unsolved problem to characterize the positive integers n for which all coefficients of the polynomials $\Phi_n(x)$ are positive.