# Math 261: Homework 6 solutions

Ch. 5

33.

(i)

$$\lim_{x\to\infty} \frac{x+\sin^3 x}{5x+6} = \lim_{x\to\infty} \frac{1+\frac{\sin^3 x}{x}}{5+\frac{6}{x}} = \frac{1}{5}.$$

(ii)

$$\lim_{x\to\infty} \frac{x\sin x}{x^2+5} = \lim_{x\to\infty} \frac{1}{x+\frac{5}{x}}.\sin x = 0$$

since $|\sin x| \leq 1$.

(iii)

$$\begin{aligned}\lim_{x\to\infty} \sqrt{x^2+x}-x &= \lim_{x\to\infty} \frac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x} \\ &= \lim_{x\to\infty} \frac{x}{\sqrt{x^2+x}+x} = \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x}}+1} = \frac{1}{2}.\end{aligned}$$

(iv) This limit does not exist. For

$$\lim_{x\to\infty} \frac{x^2(1+\sin^2 x)}{(x+\sin x)^2} = \lim_{x\to\infty} \frac{1+\sin^2 x}{(1+\frac{\sin x}{x})^2}.$$

The denominator tends to 1 but the numberator does not approach a limit as $x \to \infty$.

40. (a) You cut the regular polygon into $n$ isosceles triangles. To work out the length of the end of each of these triangles, cut it in half to get two right angles triangles, angle $\pi/n$, hypotenuse $r$. The opposite side of those has length $r\sin(\pi/n)$. Hence the overall perimeter is $2nr\sin(\pi/n)$.

(b) I'll substitute $x = \pi/n$.

$$\lim_{n\to\infty} 2nr\sin(\pi/n) = \lim_{x\to 0^+} \frac{2\pi r\sin(x)}{x}.$$

We know that this limit is $2\pi r$, the circumference of the circle.

Ch. 6

1. (ii) No, since $f(x) = 1$ for $x > 0$ and $-1$ for $x < 0$, you can't repair that jump...

(iv) No. Remember this function is continuous only at the irrational numbers.

2. IV-17: (i),(ii),(iii) are all points except integers. IV-19: (i) is all points not of the form $k/10$ for integer $k$. (ii) is all points not of the form $k/100$ for integer $k$. (iii) is no points.

5. Let $f(x) = a$ for $x$ irrational and $f(x) = x$ for $x$ rational.

13. (a) Since $f$ is continuous on $[a,b]$, the limits $\lim_{t\to a^+} f(t)$ and $\lim_{t\to a^-} f(t)$ exist. Let

$$g(x) = \begin{cases} \lim_{t\to a^+} f(t) & x \le a, \\ f(x) & a < x < b, \\ \lim_{t\to b^-} f(t) & x \ge b. \end{cases}$$

(b) Let $f(x) = \frac{1}{x-a}$.

Ch. 7

1.

(i) see text.

(ii) bounded above and below; does not attain bounds.

(iii) see text.

(iv) bounded below not above; minimum value $= 0$.

(v) Bounded above and below. It is understood that $a > -1$ so that $-a-1 < a+1$. If $-1 < a < 1/2$ then $a < -a-1$, so $f(x) = a+2$ for all $x$ in $(-a-1, a+1)$, so $a+2$ is the maximum and minimum value. If $-1/2 < a \le 0$ then $f$ has the minimum value $a^2$, and if $a \ge 0$ then $f$ has the minimum value 0. Since $(a+2) \ge (a+1)^2$ only for $[-1-\sqrt{5}]/2 < a < [1+\sqrt{5}]/2$, when $a \ge 1/2$ this function $f$ has a maximum value only for $a \le [1+\sqrt{5}]/2$ (the maximum value being $a+2$).

2. (ii) $n = -5$, since $f(-5) = 2(-5)+1 < 0 < f(-4)$.

(iv) $n = 0$ since both roots of $f(x) = 0$ lie in $[0,1]$.

Other parts: see text.

3(ii). Look at $f(x) = \sin x - x + 1$. It is continuous. And $f(0) = 1$, $f(1000) = \sin 1000 - 1000 + 1 < 0$. So by the IVT there's some $x$ (between 0 and 1000) with $f(x) = 0$.

Other parts: see text.

4(a). Take for example $(x+1)(x+2)\cdots(x+k)(x^{n-k}+1)$.