Math 261: Homework 1 (due W October 8)

Part I: reading.
Read chapters 1 and 2 from the text, try to make sense of the ideas.

Part II: axiom chasing.
Be very laborious and record each axiom (P1)–(P12) each time you use it!

1. Let a be any number. Suppose that b is a number with the property that $a + b = 0$. *Prove* using axioms (P1)–(P12) that $b = -a$. This shows: $-a$ is the *unique* number which when added to a gives zero. If you like: "additive inverses are unique".

2. Prove that "multiplicative inverses are unique". So, take $a \neq 0$ and prove that a^{-1} (as given by (P7)) is the unique number which when multiplied by a gives you 1.

3. Prove $a.(-1) = -a$ (you may assume that $a.0 = 0$ for all a).

4. Prove $(a/b)^{-1} = b/a$ for all $a, b \neq 0$.

5. Prove $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$ for all a, b, c, d with $b, c, d \neq 0$.

6. Suppose $a > b$ and $c < 0$. Prove $ac < bc$ using axioms (P1)–(P12).

Part III: questions from the book.
You no longer need to be too careful about which axioms you are using if its obvious stuff. But try to give complete proofs that make logical sense (to the grader). Complete sentences are allowed in math!!!

Ch.1 1(iii),(iv).
Ch.1 4(i),(ii),(iii),(iv).
Ch.1 5(vi)
Ch.1 7.
Ch.1 10.
Ch.1 11(i),(ii),(iii).
Ch.1 18(a),(b).
Ch.1 20.