

Winter 2007

Calculus I Practise Midterm

Name: \_\_\_\_\_

1	2	3	4	5	6	TOT.

Answer ALL questions. Each question is worth TEN points. Show all your work and try to justify your answers whenever possible – that way I can give some credit even for wrong answers.

1. True or False?

(a) If  $f(x) = e^2$  then  $f'(x) = 2e$ .

(b) If  $f$  is differentiable then  $\frac{d}{dx}(\sin(f(x))) = \cos(f(x)) \cdot f'(x)$ .

(c) If  $\lim_{x \rightarrow 6} f(x)$  exists then the limit must be  $f(6)$ .

(d) If  $f(x)$  is continuous at  $x = a$  then  $f(x)$  is differentiable at  $x = a$ .

(e) If  $f''(6)$  exists then the function  $f'(x)$  is continuous at  $x = 6$ .

2. (a) Write down the *quotient rule* as precisely as you can.

(b) Differentiate the following functions. Simplify your answer as far as you think is reasonable. Show your working to get partial credit!

(i)  $\frac{4-x}{3+x}$ .

(ii)  $x(\cos^2 x + 1)$ .

(iii)  $\tan(\sqrt{xe^x})$ .

(iv)  $\sin(\ln x)$ .

3. At which point of the curve  $y = x^2 - 3x + 7$  is the tangent line parallel to the line  $x + y = 1$ ?

4. (a) What is the definition of the function  $\arccos x$ ? What is its domain?

(b) If  $y = \arccos x$ , use implicit differentiation to calculate  $\frac{dy}{dx}$  in terms of  $x$ .

5. (a) Precisely what does it mean to say that a function  $f(x)$  is *differentiable* at  $x = a$ ?

(b) Suppose  $f$  and  $g$  are differentiable functions such that  $f(9) = 12$ ,  $f'(9) = -1$ ,  $f'(5) = 8$ ,  $f'(6) = 7$ ,  $g(9) = 6$ ,  $g'(9) = 5$ . Calculate

(i)  $(f \circ g)'(9)$ .

(ii)  $(f \cdot g)'(9)$ .

(iii)  $\left(\frac{d}{dx} \ln f(x)\right) |_{x=9}$ .

6. A particle oscillates so that its displacement  $f(t)$  from the origin at time  $t$  (in seconds) is given by the equation  $f(t) = e^{-3t} \sin t$  (in meters).

(a) How fast is the particle moving at time  $t = 0$ ? Include the units!

(b) What is the acceleration of the particle at time  $t = 0$ ? Include the units!

(c) Calculate  $\lim_{t \rightarrow \infty} f(t)$ . What does this tell you about the long term behavior of the particle?