

Winter 2007

Calculus I Practise Midterm

Name: \_\_\_\_\_

1	2	3	4	TOT.

Answer ALL questions. Each question is worth TEN points. Show all your work and try to justify your answers whenever possible – that way I can give some credit even for wrong answers.

1. Describe the *domains* of the following functions (work in radians when talking about angles for sin):

(a)  $\sin\left(\frac{1}{x}\right)$ .

(b)  $\sqrt{1-x^2}$ .

(c)  $\ln(\ln(x))$ .

(d)  $\arcsin(x+1)$ .

2. Use the transformation rules to sketch the following graphs. Label enough points (e.g. axis intercepts) on the axes so that the scale of your sketch is clear.

(a)  $f(x) = \frac{1}{x+1} + 1$ .

(b)  $f(x) = -\sin(2x)$ .

(c)  $f(x) = \begin{cases} -x & \text{if } x < 0, \\ e^x - 1 & \text{if } x \geq 0. \end{cases}$

For each of your graphs (a)–(c), indicate also any *horizontal* or *vertical* asymptotes.

3. (a) Write the sentence explaining in words what it means to say " $\lim_{x \rightarrow 7^+} f(x) = 2$ ".

(b) Write the sentence explaining in words what it means to say " $\lim_{x \rightarrow \infty} f(x) = -\infty$ ".

(c) What does it mean to say that  $f(x)$  is *continuous* at  $x = a$ ?

(d) What does it mean to say that  $f(x)$  is *continuous on the interval*  $[0, 1]$ ?

(e) Draw a rough sketch of a function that is continuous on the intervals  $[0, 1]$  and  $[1, 2]$  but not on the interval  $[0, 2]$ . (There are many answers to this!)

4. (a) Solve the following equation for  $x$ , giving your answer correct to 3 decimal places.

$$5^x = 77.$$

(b) The half-life of palladium-100 is four days. The initial mass of a sample is 2g. Write down a formula for the mass of the sample after  $t$  days. Use your formula to work out how many days it will take before the mass is reduced to 0.02g.

5. Using limit laws and any other tricks you can think of, calculate the following limits exactly. If the limit doesn't exist, write DNE. Try to justify all your answers with some working or a short explanation in words.

(a)  $\lim_{x \rightarrow 1} e^{x^3 - x}$ .

(b)  $\lim_{x \rightarrow 1} \frac{x^2 - 9}{x^2 + 2x - 3}$ .

(c)  $\lim_{t \rightarrow 2} \frac{t^2 - 4}{t^3 - 8}$ .

(d)  $\lim_{x \rightarrow 0} \frac{x}{|x|}$  (remember:  $|x|$  means the absolute value of  $x$ , so its  $x$  if  $x \geq 0$  and  $-x$  if  $x < 0$ .)

(e)  $\lim_{s \rightarrow 16} \frac{4 - \sqrt{s}}{s - 16}$ .

6. (a) State the *Intermediate Value Theorem* as precisely as you can.

(b) Let

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0, \\ 3 - x & \text{if } 0 \leq x < 3, \\ (x - 3)^2 & \text{if } x > 3. \end{cases}$$

Calculate:

(i)  $\lim_{x \rightarrow 0^+} f(x)$ .

(ii)  $\lim_{x \rightarrow 0^-} f(x)$ .

For which  $x$  is  $f(x)$  discontinuous?