Winter 2007

Calculus I Practise Final

Name:______________________________

University ID:_______________________

(Don't forget to bring your ID with you – according to university regulations you are meant to have your university ID with you for all examinations.)

1	2	3	4	5	6	7	8	TOT.

Answer ALL questions. Each question is worth TEN points. Show all your work and try to justify your answers whenever possible – that way I can give some credit even for wrong answers.

1. (a) Sketch the graph of $f(x) = \tan x$ on the domain $(-\pi/2, \pi/2)$. Indicate on your graph the *slope* of the tangent line to the curve at the origin.

(b) For $f(x)$ as in (a), use the transformation rules to sketch the graph of $g(x) = f(\frac{x}{2} + \pi)$.

(c) For $f(x)$ as in (a) and $g(x)$ as in (b), is it true that $g'(x) = f'(\frac{x}{2} + \pi)$?

2. (a) Complete the following statement of the chain rule: if f and g are differentiable functions then

$$(f \circ g)'(x) =$$

(b) The hyperbolic functions $\sinh x$ and $\cosh x$ are defined by

$$\sinh x = \frac{1}{2}(e^x - e^{-x}), \qquad \cosh x = \frac{1}{2}(e^x + e^{-x}).$$

Using these definitions, prove that $\cosh^2 x - \sinh^2 x = 1$.

(c) Use the chain rule and the definitions in (b) to show that $\frac{d}{dx}(\sinh x) = \cosh x$ and $\frac{d}{dx}(\cosh x) = \sinh x$.

(d) Let $\tanh x = \frac{\sinh x}{\cosh x}$. Use the quotient rule to prove that $\frac{d}{dx}(\tanh x) = \frac{1}{\cosh^2 x}$.

3. Differentiate the following functions, simplifying your answers as far as you judge is reasonable.

(a) $\sin^2 x$.

(b) 2^x.

(c) $\ln(\arctan x)$.

(d) $\sin x \cdot \cos x$.

(e) $\frac{x^2+1}{(x+1)^2}$.

4. Calculate the following limits using any method you like. If the limit is ∞ or $-\infty$, say so. If the limit does not exist, write DNE.

(a) $\lim_{x\to 0} \frac{\sin 2x}{x}$.

(b) $\lim_{x\to 2} \frac{x^2-2x+1}{x^3-9x^2+3}$.

(c) $\lim_{x\to \infty} \frac{e^{3x}-2-3x}{x^2}$.

(d) $\lim_{x\to 0^+} x^2 \ln x$.

(e) $\lim_{v\to 4^-} \frac{4-v}{|4-v|}$.

5. Determine the constant a such that the function $f(x) = x^2 + \frac{a}{x}$ has a local minimum at $x = 2$. Does the function have a global maximum anywhere (for any value of a)?

6. Find local maxima, minima and points of inflection for the function $f(x) = x^2 e^{5x}$. Sketch the graph.

7. A farmer wants to fence an area of 1.5 million square feet in a rectangular field, then divide it in half with a fence parallel to one of the sides of the rectangle. How should he do this so as to minimize the cost of fence?

8. True or False? Justify your answers briefly if you can.
(a) If $f'(c) = 0$ then f has a local maximum or minimum at c.

(b) If f has a global maximum at c then $f'(c) = 0$.

(c) If $f'(x) = g'(x)$ for all x then $f(x) = g(x)$.

(d) There exists a function $f(x)$ such that $f(x) < 0$, $f'(x) < 0$ and $f''(x) > 0$ for all x.

(e) There exists a function $f(x)$ such that $f(x) > 0$, $f'(x) < 0$ and $f''(x) > 0$ for all x.