Winter term, 1067

Discrete Mathematics II Midterm

Name:____________________________

1	2	3	4	5	TOT.

Answer ALL questions. Each question is worth FIVE points. Show all your work and show your working – even if you give the correct answer you will not get full marks without it.

1. An undirected graph $G = (V, E)$ is called *regular of degree d* if it has no loops and all its vertices have degree d.

(a) Give an example of two non-isomorphic regular graphs of degree 3 with $|V| = 10$.

(b) How many regular graphs of degree 4 with $|V| = 5$ are there up to isomorphism?

(c) How many regular graphs of degree 3 with $|V| = 5$ are there up to isomorphism?

(d) How many regular graphs of degree 2 with $|V| = 5$ are there up to isomorphism?

2. Let G be an undirected graph with $n > 1$ vertices such that $G \cong \overline{G}$.
(a) Prove that $n = 4k$ or $n = 4k + 1$ for some integer $k \geq 1$.

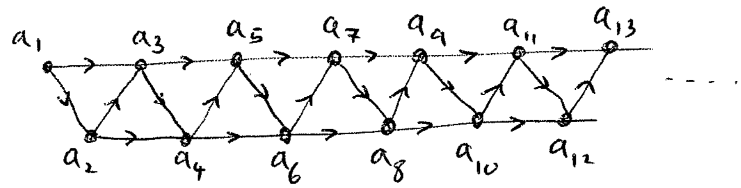
(b) Give an example of such a graph G for which $n = 4$ and one for which $n = 5$.

3. Let G be an undirect graph with no loops.
(a) What is an *Euler circuit* and what is a *Hamilton cycle*?

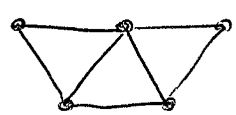
(b) How do you tell which G have an Euler circuit? (No proof needed...)

(c) Let Q_4 be the graph of the 4-dimensional hypercube, with vertex set $V = \{b_1b_2b_3b_4 | \text{for all binary digits } b_1, b_2, b_3, b_4 = 0 \text{ or } 1\}$ and an edge joining two vertices if they differ in just one digit.
List the vertices of Q_4 in an order that corresponds to a Hamilton path in Q_4 starting at 0000 and ending at 1000.

5. (a) In the following graph, how many directed paths are there from a_1 to a_n?

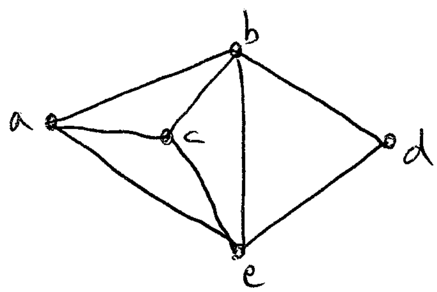


(b) How many different spanning subgraphs does the following graph have?



How many of those are spanning trees?

4. Let G be the following graph:



(a) What is the degree of vertex e?

(b) What is the distance from a to d?

(c) If I add an edge $\{d, c\}$ to the graph, is the resulting graph a planar graph? If not, why not? If so, demonstrate...

(d) Write down the adjacency matrix of G (labelling its rows and columns a, b, c, d, e in order):

	a	b	c	d	e
a					
b					
c					
d					
e					