

Fall term, 1066

Discrete Mathematics I Midterm

Name: \_\_\_\_\_

1	2	3	4	5	TOT.

Answer ALL questions. Each question is worth THREE points. Show all your work and show your working – even if you give the correct answer you will not get full marks without it.

1. (a) How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 100$$

with  $x_1, x_2, x_3, x_4 \geq 0$ ?

(b) How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 100$$

with  $x_1, x_2, x_3, x_4 > 0$ ?

(c) How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 \leq 100$$

with  $x_1, x_2, x_3, x_4 \geq 0$ ?

2. In this question,  $A$ ,  $B$  and  $C$  are arbitrary sets. You should remember the definitions of the sets  $A \setminus B$  (“set minus”) and  $A \Delta B$  (“symmetric difference”)...

(a)  $A \Delta B = (A \setminus B) \cup (B \setminus A)$ . True or False? Justify your answer by giving a proof or a counterexample.

(b)  $(A \Delta B) \setminus C = A \Delta (B \setminus C)$ . True or False? Justify your answer by giving a proof or a counterexample.

(c)  $(A \setminus B) \Delta C = A \setminus (B \Delta C)$ . True or False? Justify your answer by giving a proof or a counterexample.

3. Answer the following True or False, justifying your answers carefully.

(a)  $(p \wedge (\neg q)) \rightarrow (p \rightarrow q)$  is a tautology.

(b) If  $x$  is a real number with the property that  $x + y$  is irrational for every irrational number  $y$ , then  $x$  is rational.

(c) If  $n$  is an integer such that  $n^2$  is divisible by 4 then  $n$  is divisible by 4.

(d)  $\sqrt{5}$  is rational.

(e) If  $x$  is rational and  $y$  is irrational then  $x \cdot y$  is irrational.

4. (a) How many arrangements are there of the word ABRACADABRA?  
How many of those involve no pair of consecutive A's?

(b) Find simple expressions for each of the following:

$$\bullet \sum_{i=1}^{10} (2i - 1).$$

$$\bullet \sum_{i=0}^n \binom{n}{i} 2^i.$$

$$\bullet \sum_{i=1}^n \binom{n}{i} 2^i.$$

5. True or False? If True give a proof, if False give a counterexample. (You might not need to draw out the full truth table to do this!)

(a) If  $(q \wedge r) \Rightarrow p$  and  $q \Rightarrow \neg r$  then  $p$  is true.

(b) If  $q \vee \neg r$  and  $\neg(r \rightarrow q) \rightarrow \neg p$  are both true then  $p$  is true.

(c) If  $p \Rightarrow (q \vee r)$ ,  $q \Rightarrow s$  and  $r \Rightarrow \neg p$  then  $p \Rightarrow s$ .