Homework 6 (due Wednesday November 8 despite the midterm)

- There is a midterm in class next Tuesday. Attached to this homework is a practise midterm – making sure you can do all of that is one of the best ways to prepare for the midterm. As usual I will post the solutions to the practise midterm on my web page later this week.

- So far in class we've covered sections 1.1-1.5, 1.7, 4.1, 4.7 and 2.1-2.4 in the text book. All that material is fair game for the midterm! However, the midterm will *mainly* focus on the recent stuff from chapter 2 (logical connectives, truth tables, proof techniques) and sections 1.4 (sets), 1.5,1.7 (functions). Just don't forget all the earlier stuff too (prime numbers, rational and irrational numbers, division algorithm, Euclidean algorithm, fundamental theorem of arithmetic).

- For homework this week, do the following problems:

  - Section 1.5: 5,6,13.
  - Section 1.7: 1,3,4,5,7,13,15.
  - Supplementary exercises to chapter 2: 12.
  - Section 3.1: 1.

  You should definitely do this homework BEFORE the midterm – especially since you haven't done any problems on functions yet and there will be questions on the midterm about the definitions of "1–1", "onto" and "1–1 correspondence".

- BAD NEWS. Our final exam is at 3.15pm on Friday December 8. I've been told that I am absolutely not allowed to give the final exam to anyone any earlier than this scheduled time. Sorry!!!! (Look on the bright side – it will give you plenty of time to revise for it.)

1. Let p, q and r be propositions.

(a) Calculate the truth table for the compound proposition $((p \vee q) \wedge r) \rightarrow ((p \wedge q) \vee r)$.

(b) Is $((p \vee q) \wedge r) \rightarrow ((p \wedge q) \vee r)$ a tautology?

(c) $((p \vee q) \wedge r) \Leftrightarrow ((p \wedge q) \vee r)$. True or False?

2. Let n be an integer. Prove carefully that n is a multiple of 3 if and only if $n^2 - 3n + 2$ is not divisible by 3.

3. True or False? If True give a proof, if False give a counterexample.

(a) If n is an integer such that n^2 is divisible by 4 then n is divisible by 4.

(b) If n is an integer such that n^2 is divisible by 5 then n is divisible by 5.

(c) $\sqrt{5}$ is rational.

(d) If x is rational and y is irrational than $x \cdot y$ is irrational.

4. Let $f : S \to T$ be a function. Write down precise definitions of the following:

(a) f is 1–1.

(b) f is onto.

Now consider the following functions. For each of them work out whether they are 1–1, onto or both. Explain your answer.

(c) $f : \mathbb{R} \to \mathbb{R}, x \mapsto 10^x$.

(d) $f : \mathbb{P} \times \mathbb{P} \to \mathbb{P}, (x, y) \mapsto GCD(x, y)$.

(e) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}, (x, y) \mapsto (x + y, x - y)$.

5. True or False? If True give a proof, if False give a counterexample. (You might not need to draw out the full truth table to do this!)
(a) If $(q \wedge r) \rightarrow p$ and $q \rightarrow \neg r$ then p.

(b) If $q \vee \neg r$ and $\neg(r \rightarrow q) \rightarrow \neg p$ then p.

(c) (Hard) If $p \rightarrow (q \vee r)$, $q \rightarrow s$ and $r \rightarrow \neg p$ then $p \rightarrow s$.