Fall 2006

Discrete Mathematics I Practise Midterm

Name: J. BRUNDAN

1	2	3	4	5	TOT.
3	3	3	3	3	15

Answer ALL questions. Each question is worth THREE points. Show all your work and show your working – even if you give the correct answer you will not get full marks without it.

The REAL MIDTERM is next Tuesday in class. I will post the solutions to this practise midterm on my web page

`http://uoregon.edu/~brundan/math231fall06/hw.html`

by the end of Friday this week. Like this practise, the midterm will be based on the material covered in class so far, which is roughly sections 1.1,1.2,1.3,1.4, 4.1 and bits of 4.7 in the text book.

inclusive!

1. How many multiples of 7 are there between −74 and 167?

$$\left\lfloor \frac{167}{7} \right\rfloor - \left\lfloor \frac{-75}{7} \right\rfloor = 23 - (-11) = 23 + 11 = \underline{\underline{34}}$$

2.

(a) If $m \in \mathbb{P}$ and $n \in \mathbb{Z}$, write down as precisely and succinctly as you can the *definitions* of the numbers n MOD m and n DIV m.

If $n = qm + r$ with $0 \leq r < m$

then n MOD $m = r$, n DIV $m = q$.

(b) Find $GCD(51, 43)$.

$51 = 1 \cdot 43 + 8$
$43 = 5 \cdot 8 + 3$
$8 = 2 \cdot 3 + 2$
$3 = 1 \cdot 2 + 1$ $\leftarrow$ $GCD(51, 43) = 1$

(c) Find integers s and t such that $43s + 51t = GCD(51, 43)$.

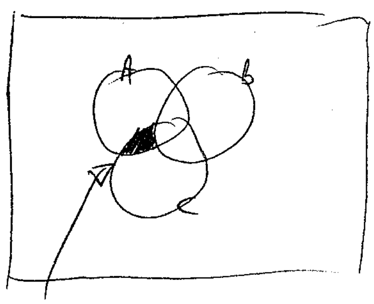
$$1 = 3 - 2 = 3 - (8 - 2 \cdot 3) = 3 \cdot 3 - 8$$
$$= 3 \cdot (43 - 5 \cdot 8) - 8 = 3 \cdot 43 - 16 \cdot 8$$
$$= 3 \cdot 43 - 16 \cdot (51 - 43)$$

$$1 = \underbrace{19}_{s=19} \cdot 43 \underbrace{-16}_{t=-16} \cdot 51$$

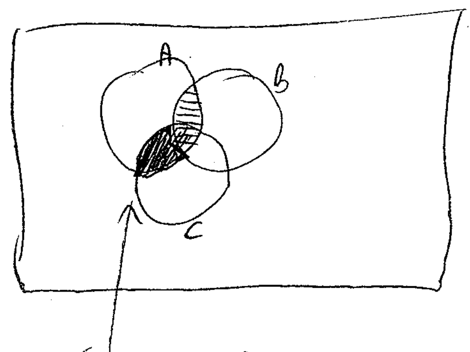
$s = 19$

$t = -16$

3. Is it true for arbitrary sets A, B and C that $(A \backslash B) \cap C = (A \cap C) \backslash (B \cap C)$? Explain.



$(A-B) \cap C$



$(A \cap C) - (B \cap C)$

Yes its true – the Venn diagrams show it.

4. Consider the loop

```
while n < 200 do
    n := 18 − 7n
od
```

Which of the following are loop invariants?

(a) n is even.

If n is even then $18 - 7n =$ even − even is even

<u>Yes</u>

(b) n is odd.

If n is odd then $18 - 7n =$ even − odd is odd

<u>Yes</u>

(c) n is divisible by 4.

<u>No</u> eg take $n = 0$ at start (div. by 4)

at end have $n = 18$ (not div. by 4)

5. Here is a pseudo-computer program.
Input: a positive integer n.

```
begin
    m := n
    k := 0
    while m is even do
        m := m/2
        k := k + 1
    od
    return k and m
end
```

What does it do? That is, write a sentence specifying the outputs k and m as precisely as you can.

It returns an odd integer m and $k \geq 0$ such that $n = 2^k \cdot m$.

(Proof: $k \geq 0$ and $n = 2^k \cdot m$ are loop invariants.
True at start $\therefore$ true at end, when m is odd
$\therefore$ At end m is odd, $k \geq 0$ and $n = 2^k \cdot m$ //)

The question didn't ask for this ...