Homework # 25. Due to Wednesday, May 27, 11:00 am

(1) Prove that any $K(\mathbf{Z}_2, n)$ is infinite dimensional space for each $n \geq 1$.

(2) Let M be a simply-connected compact closed manifold with $\dim M = 3$. Prove that M is homotopy equivalent to S^3.

(3) Construct a space X with $H_*(X) = H_*(pt)$, which is not homotopy equivalent to a point.

(4) Let $h : S^3 \to S^2$ be the Hopf map. Let $\lambda \geq 1$ be an integer. Define a map

$$f_\lambda : S^3 \xrightarrow{h} \underbrace{S^3 \vee \cdots \vee S^3}_{\lambda} \xrightarrow{h \vee \cdots \vee h} S^2.$$

Prove that the space $X_\lambda = S^2 \cup_{f_\lambda} D^4$ is homotopy equivalent to a closed compact manifold of dimension four if and only if $\lambda = 1$.

(5) Let $D^3 \subset T^3$ and $c : T^3 \to S^3$ be a map which collapses a complement of $D^3 \subset T^3$ to a point. Prove that the map $g : T^3 \xrightarrow{c} S^3 \xrightarrow{h} S^2$ (where $h : S^3 \to S^2$ is the Hopf map) induces trivial homomorphism on homology and homotopy, but is not homotopic to a constant map.