

Homework # 24. Due to Wednesday, May 20, 11:00 am

- (1) Let M be a compact closed manifold of dimension $\dim M = 2k$. Prove that if $H_{k-1}(M; \mathbf{Z})$ is torsion-free, then the group $H_k(M; \mathbf{Z})$ is also torsion-free.
- (2) Let M be a compact manifold with boundary $\partial M \neq \emptyset$. Prove that M does not retract to its boundary ∂M .
- (3) Show that if the groups $\tilde{H}^n(X; \mathbf{Q})$ and $\tilde{H}^n(X; \mathbf{Z}_p)$ are zero for all n and all primes p , then $\tilde{H}^n(X; \mathbf{Z}) = 0$ for all n .
- (4) Let W be a compact manifold with boundary $\partial W = M$. Prove that the Euler characteristic $\chi(M)$ is even.
- (5) Read on the Poincaré duality for manifolds with boundary (see the link). Let W be an oriented manifold with $\partial W = M$, $\dim W = 4k + 1$. Prove that the signature $\sigma(M) = 0$.