

Homework # 21. Due to Wednesday, April 29, 11:00 am

(1) Assuming the fact that $H^*(\mathbf{RP}^n; \mathbf{Z}/2) = \mathbf{Z}/2[x]/(x^{n+1})$, show that there is no map

$$f : \mathbf{RP}^n \rightarrow \mathbf{RP}^k$$

which induces a non-trivial homomorphism $f^* : H^1(\mathbf{RP}^k; \mathbf{Z}/2) \rightarrow H^1(\mathbf{RP}^n; \mathbf{Z}/2)$ if $n > k$.

(2) Let $f : X \rightarrow Y$ be a map, and

$$f_* : H_*(X; R) \rightarrow H_*(Y; R), \quad f^* : H^*(Y; R) \rightarrow H^*(X; R)$$

be the induced homomorphisms. Prove that

$$f_*(\sigma \cap f^*(\phi)) = f_*(\sigma) \cap \phi, \quad \sigma \in H_*(X; R), \quad \phi \in H^*(Y; R).$$

(3) Let M_g^2 be the oriented surface of the genus g . Let $[M_g^2] \in H_2(M_g^2; \mathbf{Z}) \cong \mathbf{Z}$ be a generator. Define the homomorphism $D : H^1(M_g^2; \mathbf{Z}) \rightarrow H_1(M_g^2; \mathbf{Z})$ by the formula $D : \alpha \mapsto [M_g^2] \cap \alpha$. Compute the homomorphism D .

(4) Let N_g^2 be the non-oriented surface of the genus g , i.e.

$$N_g^2 = T^2 \# \dots \# T^2 \# \mathbf{RP}^2.$$

Let $[N_g^2] \in H_2(N_g^2; \mathbf{Z}/2) \cong \mathbf{Z}/2$ be a generator. Define the homomorphism

$$D_2 : H^1(N_g^2; \mathbf{Z}/2) \rightarrow H_1(N_g^2; \mathbf{Z}/2)$$

by the formula $D_2 : \alpha \mapsto [N_g^2] \cap \alpha$. Compute the homomorphism D_2 .