

Homework # 20. Due to Wednesday, April 22, 11:00 am

- (1) Compute the cup-product for $H^*(\mathbf{RP}^2; \mathbf{Z}/2)$.
- (2) Let N_g^2 be nonoriented surface of genus g , i.e. $N_g^2 = T^2 \# \dots T^2 \# \mathbf{RP}^2$. Compute the cup-product for $H^*(N_g^2; \mathbf{Z}/2)$.
- (3) Compute the cup product for $H^*(\mathbf{RP}^2; \mathbf{Z}/2^k)$, $k \geq 2$.
- (4) Let $X = S^1 \cup_{f_k} e^2$, where $f_k : S^1 \rightarrow S^1$ is a degree k map. Compute the cup product for $H^*(X; \mathbf{Z}/k)$.
- (5) Let $T : \mathbf{R}^{q+1} \rightarrow \mathbf{R}^{q+1}$ be given by a linear isomorphism sending vertices (v_0, \dots, v_q) to (v_q, \dots, v_0) respectively. It gives a chain map $t : \mathcal{C}_*(X) \rightarrow \mathcal{C}_*(X)$ sending each generator $f : \Delta^q \rightarrow X$ to the generator $\bar{f} : \bar{\Delta}^q \xrightarrow{T} \Delta^q \xrightarrow{f} X$. Prove that there is a chain homotopy between t and $(-1)^{\frac{q(q+1)}{2}} Id$.
- (6) Prove that any map $f : S^4 \rightarrow S^2 \times S^2$ must induce the zero homomorphism

$$f_* : H_4(S^4) \rightarrow H_4(S^2 \times S^2).$$