Homework # 1. Due to Wednesday, April 15, 11:00 am

(1) Let $0 \le p, q \le n-1$, and the wedge $S^p \vee S^q$ is embedded to S^n. Compute the homology groups $H_q(S^n \setminus (S^p \vee S^q))$.

(2) Prove that for each $n \ge 1$ there exists a space X with

$$\widetilde{H}_q(X) = \begin{cases} \mathbf{Z}/m, & \text{if } q = n, \\ 0, & \text{if } q \ne n. \end{cases}$$

(3) Let $\mathcal{H} = \{H_q\}$ be a graded abelian group. We assume that $H_q = 0$ for $q < 0$, and H_0 is a free abelian. Prove that there exists a space X such that $H_q(X) = H_q$ for all q. In particular, construct a space X with the homology groups:

$$\widetilde{H}_q(X) = \begin{cases} \mathbf{Z}[\frac{1}{p}], & \text{if } q = n, \\ 0, & \text{if } q \ne n. \end{cases}$$

(4) Let $X = \mathbf{RP}^{2n_1} \vee \cdots \vee \mathbf{RP}^{2n_s}$. Prove that any map $f : X \to X$ has a fixed point.

(5) Consider the subset $X \subset \mathbf{R}^2$ consisting of two circles C_+ and C_- of radius 1 centered at $(0, +1)$ and $(0, -1)$ respectively. Let $f : X \to X$ be a reflection in the y-axis on C_+ and a degree two map on C_-. Show that $\mathrm{Lef}(f) = 0$, however any map $g : X \to X$ homotopic to f has a fixed point.