Homework # 17. Due to Wednesday, March 11, 11:00 am

(1) Prove that the group $\mathrm{Tor}(G,H)$ is well-defined, i.e. it does not depend on the choice of resolution.

(2) Let G,H be abelian groups. Prove that there is an isomorphism $\mathrm{Tor}(G,H)\cong\mathrm{Tor}(H,G)$.

(3) Let G be an abelian group. Denote $T(G)$ a maximal torsion subgroup of G. Show that $\mathrm{Tor}(G,H)\cong T(G)\otimes T(H)$ for finite generated abelian groups G,H. Give an example of abelian groups G,H, so that $\mathrm{Tor}(G,H)\neq T(G)\otimes T(H)$.

(4) Prove that the following sequence is exact:

$$0\longleftarrow \mathrm{Coker}\ \beta^{\#}\longleftarrow \mathrm{Hom}(R,H)\xleftarrow{\beta^{\#}}\mathrm{Hom}(F,H)\xleftarrow{\alpha^{\#}}\mathrm{Hom}(G,H)\longleftarrow 0.$$

(5) Prove that $\mathrm{Ext}(\mathbf{Z},H)=0$ for any group H.

(6) Prove the isomorphisms: $\mathrm{Ext}(\mathbf{Z}/m,\mathbf{Z}/n)\cong\mathbf{Z}/m\otimes\mathbf{Z}/n$, $\mathrm{Ext}(\mathbf{Z}/m,\mathbf{Z})\cong\mathbf{Z}/m$.

(7) Let X be a space so that the groups $H_q(X)$ are finitely generated for all $q\geq 0$. Prove that $H^q(X;\mathbf{Z})$ are also finitely generated and $H^q(X;\mathbf{Z})\cong F(H_q(X;\mathbf{Z}))\oplus T(H_{q-1}(X;\mathbf{Z}))$.

(8) Let F be a field. Prove that

$$H^q(X;F)=\mathrm{Hom}_F(H_q(X;F),F).$$

(9) Let F be a free abelian group. Show that $\mathrm{Ext}(G,F)=0$ for any abelian group G.

(10) Give a detailed proof of the following
Theorem. *Let X be a space, and G an abelian group. Then there is a split exact sequence*

$$0\longrightarrow H^q(X;\mathbf{Z})\otimes G\longrightarrow H^q(X;G)\longrightarrow \mathrm{Tor}(H^{q+1}(X;\mathbf{Z}),G)\longrightarrow 0$$

for any $q\geq 0$. Again the splitting is not natural.