Homework # 15. Due to Wednesday, February 26, 11:00 am

(1) Prove that $H_0(X) \cong \mathbf{Z} \oplus \ldots \oplus \mathbf{Z}$, where the number of $\mathbf{Z}$'s is the same as the number of path-connected components of X.

(2) Let $0 \longrightarrow \mathcal{C}' \xrightarrow{i} \mathcal{C} \xrightarrow{j} \mathcal{C}'' \longrightarrow 0$ be a short exact sequence of complexes. Prove that the homomorphism $\partial : H_q(\mathcal{C}'') \to H_{q-1}(\mathcal{C}')$ is well-defined.

(3) Prove the exactness of

$$\cdots \longrightarrow H_q(\mathcal{C}') \xrightarrow{i_*} H_q(\mathcal{C}) \xrightarrow{j_*} H_q(\mathcal{C}'') \xrightarrow{\partial} H_{q-1}(\mathcal{C}') \xrightarrow{i_*} \cdots$$

at the term $H_q(\mathcal{C}')$.

(4) Let $B \subset A \subset X$ be a triple of spaces. Prove that the sequence of complexes

$$0 \longrightarrow \mathcal{C}(A,B) \xrightarrow{i_\#} \mathcal{C}(X,B) \xrightarrow{j_\#} \mathcal{C}(X,A) \longrightarrow 0$$

is exact.

(5) Let $f : (X,A) \to (X',A')$ be a map of pairs. Assume that the induced maps $f : X \to X'$ and $f|_A : A \to A'$ are homotopy equivalences. Prove that $f_* : H_q(X,A) \to H_q(X',A')$ is an isomorphism for each q.

(6) Exercise 12.12 from the notes.

(7) Exercise 12.13 from the notes.

(8) Exercise 12.14 from the notes.