

Homework # 13. Due to Wednesday, February 12, 11:00 am

- (1) Consider the map

$$g : S^{2n-2} \times S^3 \xrightarrow{\text{projection}} (S^{2n-2} \times S^3)/(S^{2n-2} \vee S^3) = S^{2n+1} \xrightarrow{\text{Hopf}} \mathbf{CP}^n.$$

Prove that g induces trivial homomorphism in homotopy groups, however g is not homotopic to a constant map.

- (2) Recall the following result:

Theorem 11.10. *Let X be a Hausdorff topological space. There exists a CW -complex K and a weak homotopy equivalence $f : K \rightarrow X$. The CW -complex K is unique up to homotopy equivalence.*

Prove that the CW -complex K we constructed in the proof of Theorem 11.10 is unique up to homotopy.

- (3) Let X, Y be two weak homotopy equivalent spaces. Prove that there exist a CW -complex K and maps $f : K \rightarrow X, g : K \rightarrow Y$ which both are weak homotopy equivalences.
- (4) Let $X = K(\pi, n)$. Prove that $\Omega X = K(\pi, n-1)$.
- (5) Prove that an Eilenberg-McLane space of the type $K(\pi, n)$ is unique up to weak homotopy equivalence.
- (6) Let $X = S^2$. Prove that $X|_3 = S^3$.
- (7) Let $X = \mathbf{CP}^n$. Prove that $X|_3 = X|_{2n+1} = S^{2n+1}$.