Homework # 10. Due to Wednesday, January 22, 11:00 am

(1)(60^*) Give a detailed proof that the suspension homomorphism

$$\Sigma : \pi_q(S^n) \longrightarrow \pi_{q+1}(S^{n+1})$$

is injective for $q < 2n - 1$.[1]

(2) (10^*) Prove that $\pi_n(S^n) \cong \mathbf{Z}$ for each $n \geq 1$.

(3) (10^*)Prove that $\pi_3(S^2) \cong \mathbf{Z}$, and the Hopf map $S^3 \longrightarrow S^2$ is a representative of the generator of $\pi_3(S^2)$.

(4) (10^*) Prove that two maps $f, g : S^n \longrightarrow S^n$ are homotopic if and only if $\deg f = \deg g$.

(5) (10^*) Prove that a map $f : S^n \longrightarrow S^n$, $\deg f = \lambda$ induces the homomorphism $f_* : \pi_n(S^n) \longrightarrow \pi_n(S^n)$ which is a multiplication by λ.

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[1]This is a long exercise, please take your time to write is carefully