

Homework # 9. Due to Wednesday, January 15, 11:00 am

- (1) Prove in detail that the Hopf bundle $H : S^{4n+3} \rightarrow \mathbf{HP}^n$ is locally trivial and non-trivial bundle.
- (2) Let $p : TS^2 \rightarrow S^2$ be the tangent bundle. Prove that it is non-trivial bundle.¹
- (3) Let (X, A) be a Borsuk pair, $x_0 \in A$, and (Y, y_0) an arbitrary space with a base point. Prove that the map

$$p : \mathcal{C}((X, x_0), (Y, y_0)) \rightarrow \mathcal{C}((A, x_0), (Y, y_0)), \quad p : (f : X \rightarrow Y) \mapsto (f|_A : A \rightarrow Y)$$

is a Serre fiber bundle.

- (4) Prove that finite CW -complexes X and Y are weak homotopy equivalent if and only if they are homotopy equivalent.

In the next exercises use an exact sequence in homotopy for fiber bundles.

- (5) Prove that $\pi_q(S^2) \cong \pi_q(S^3)$ for $q \geq 3$.
- (6) Compute the homotopy groups $\pi_q(\mathbf{CP}^\infty)$ for all $q \geq 0$.
- (7) Let $p : E \rightarrow B$ be a Serre bundle with the fiber F . Assume that the groups $\pi_q B$, $\pi_q F$ are finite (finitely generated) for all q . Prove that then $\pi_q E$ are finite (finitely generated) for all q .¹
- (8) Let $p : E \rightarrow B$ be a Serre bundle. Assume this bundle has a section. Prove that $\pi_q(E) \cong \pi_q B \oplus \pi_q F$.
- (9) Give details of the proof that any map $f : X \rightarrow Y$ is homotopy equivalent to a Serre fibre bundle.

¹ It requires some research on the subject. Please do it!