Homework # 4. Due to Wednesday, November 6, 11:00 am

(1) (5^*) Construct an orthogonal transformation $T_{u,v} : \mathbf{R}^n \to \mathbf{R}^n$ such that
  (a) $T_{u,v}(u) = v$;
  (b) $T_{u,v}(w) = w$ if $w \in \langle u, v\rangle^{\perp}$.

(2) (5^*) Prove that the transformation $T_{u,v}$ from (1) has the following properties:
  (i) $T_{u,u} = Id$;
  (ii) $T_{v,u} = T_{u,v}^{-1}$;
  (iii) a vector $T_{u,v}(x)$ depends continuously on u, v, x;
  (iv) $T_{u,v}(x) = x \pmod{\mathbf{R}^j}$ if $u, v \in \mathbf{R}^j$.

(3) (5^*) Construct a cellular decomposion of the product $Z = S^n \times S^k$.

(4) (5^*) Construct a cellular decomposion of the wedge $X = \Sigma(S^n \vee S^k)$. Prove the homotopy equivalence
$$\Sigma(S^n \vee S^k) \sim S^{n+1} \vee S^{k+1}.$$

(5) (80^*) Give a detailed construction of a CW-structure of the complex Grassmanian $G_k(\mathbf{C}^n)$.
Hint: You may follow and refer to the constructions given for the real Grassmannian and provide the details if the constructions/arguments are actually differ from the real case.