Homework # 4. Due to Wednesday, November 28, 9:00 am

(1) Prove the isomorphism $\pi_q(X \times Y, x_0 \times y_0) \cong \pi_q(X, x_0) \times \pi_q(Y, y_0)$.

(2) Compute $\pi_1(M_g^2)$ for two-dimensional oriented closed manifold of genus g, the sphere with g handles. Prove that the groups $\pi_1(M_g^2)$ are not isomorphic for different g.

(3) Compute $\pi_1(N_g^2(1))$ and $\pi_1(N_g^2(2))$, where

$$N_g^2(1) = T^2 \# \cdots \# T^2 \# \mathbf{RP}^2, \quad N_g^2(2) = T^2 \# \cdots \# T^2 \# K\ell^2.$$

Prove that the groups $\pi_1(N_g^2(1))$ and $\pi_1(N_g^2(2))$ are all not isomorphic to each other for different g.

(4) Compute $\pi_1(SO(n))$ for each $n \geq 2$.

(5) Compute $\pi_1(G(4,2))$.

(6) Compute $\pi_1(\mathbf{RP}^2 \setminus \{k \text{ distinct points}\})$, $k \geq 1$.

(7) Let $p : T \to X$ be a covering, and $f, g : Z \longrightarrow T$ be two maps so that $p \circ f = p \circ g$, where Z is path-connected. Assume that $f(z) = g(z)$ for some point $z \in Z$. Prove that $f = g$.

(8) Prove that $\pi_k(\mathbf{RP}^n) = 0$ if $1 < k < n$.

(9) Prove that any map $f : \mathbf{RP}^2 \to S^1$ is homotopic to a constant map.

(10) Let Kl^2 be the Klein bottle. Construct two-folded covering space $T^2 \to Kl^2$. Compute $\pi_n(Kl^2)$ for all n.

(11) Let M^2 be a non-oriented surface, and $M^2 \neq \mathbf{RP}^2$. Compute $\pi_n(M^2)$ for all $n \geq 2$.

(12) Do Exercise 7.8 from the notes.

(13) Do Exercise 7.9 from the notes.

(14) Do Exercise 7.10 from the notes.