

**Homework # 3. Due to Wednesday, October 30, 11:00 am**

- (0) Construct a homeomorphism  $SO(4) \cong SO(3) \times S^3$ .<sup>1</sup>
- (1) Prove that Definitions HE-I and HE-III are equivalent.
- (2) Give definition of a contractible space. Prove that  $\mathcal{E}(X, x_0)$  is contractible.
- (3) Prove that a space  $X$  is contractible if and only if every map  $f : Y \rightarrow X$  is null-homotopic.
- (4) Prove that a space  $X$  is contractible if and only if it is homotopy equivalent to a point.
- (5) Prove that if a subspace  $A$  is a deformation retract of  $X$  then the inclusion  $A \rightarrow X$  is a homotopy equivalence.
- (6) Let  $X, Y$  be pointed spaces. Let  $X \sim Y$ . Prove that  $\Sigma X \sim \Sigma Y$  and  $\Omega X \sim \Omega Y$ .
- (7) Prove that a  $CW$ -complex compact if and only if it is finite.
- (8) Construct a cellular decomposition of  $S^n, D^n, \mathbf{RP}^n, \mathbf{CP}^n, \mathbf{HP}^n$ .
- (9) Construct a cellular decomposition of the oriented 2-manifold of genus  $g$ .
- (10) Prove that a finite  $CW$ -complex can be embedded into Euclidian space of finite dimension.

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<sup>1</sup>If you have not done this in the homework # 2