Algebraic Topology, 2012/2013, Boris Botvinnik

Homework # 3. Due to Wednesday, October 31, 9:00 am

(1) Prove that Definitions H-I and H-III are equivalent.

(2) Give definition of a contractible space. Prove that $\mathcal{E}(X, x_0)$ is contractible.

(3) Prove that a space X is contractible if and only if every map $f : Y \longrightarrow X$ is null-homotopic.

(4) Prove that a space X is contractible if and only if it is homotopy equivalent to a point.

(5) Prove that a subspace A is a deformation retract of X if and only if the inclusion $A \longrightarrow X$ is a homotopy equivalence.

(6) Let X, Y be pointed spaces. Let $X \sim Y$. Prove that $\Sigma X \sim \Sigma Y$ and $\Omega X \sim \Omega Y$.

(7) Prove that a CW-complex compact if and only if it is finite.

(8) Construct a cellular decomposition of S^n, D^n, $\mathbf{RP}^n$, $\mathbf{CP}^n$, $\mathbf{HP}^n$.

(9) Construct a cellular decomposition of the oriented 2-manifold of genus g.

(10) Prove that a finite CW-complex can be embedded into Euclidian space of finite dimension.

(11) Construct an orthogonal transformation $T_{u,v} : \mathbf{R}^n \longrightarrow \mathbf{R}^n$ such that

- **(a)** $T_{u,v}(u) = v$;
- **(b)** $T_{u,v}(w) = w$ if $w \in \langle u, v\rangle^{\perp}$.

(12) Prove that the transformation $T_{u,v}$ from (11) has the following properties:

- **(i)** $T_{u,u} = Id$;
- **(ii)** $T_{v,u} = T_{u,v}^{-1}$;
- **(iii)** a vector $T_{u,v}(x)$ depends continuously on u, v, x;
- **(iv)** $T_{u,v}(x) = x \pmod{\mathbf{R}^j}$ if $u, v \in \mathbf{R}^j$.

(13) Do Exercise 4.12 from the notes.

(14) Do Exercise 4.13 from the notes.

(15*) Do Exercise 4.14 from the notes.