Homework # 3. Due to Wednesday, October 31, 9:00 am

(1) Prove that Definitions H-I and H-III are equivalent.

(2) Give definition of a contractible space. Prove that $E(X, x_0)$ is contractible.

(3) Prove that a space $X$ is contractible if and only if every map $f: Y \to X$ is null-homotopic.

(4) Prove that a space $X$ is contractible if and only if it is homotopy equivalent to a point.

(5) Prove that a subspace $A$ is a deformation retract of $X$ if and only if the inclusion $A \to X$ is a homotopy equivalence.

(6) Let $X$, $Y$ be pointed spaces. Let $X \sim Y$. Prove that $\Sigma X \sim \Sigma Y$ and $\Omega X \sim \Omega Y$.

(7) Prove that a $CW$-complex compact if and only if it is finite.

(8) Construct a cellular decomposition of $S^n$, $D^n$, $RP^n$, $CP^n$, $HP^n$.

(9) Construct a cellular decomposition of the oriented 2-manifold of genus $g$.

(10) Prove that a finite $CW$-complex can be embedded into Euclidian space of finite dimension.

(11) Construct an orthogonal transformation $T_{u,v}: \mathbb{R}^n \to \mathbb{R}^n$ such that

   (a) $T_{u,v}(u) = v$;

   (b) $T_{u,v}(w) = w$ if $w \in \langle u, v \rangle ^\perp$.

(12) Prove that the transformation $T_{u,v}$ from (11) has the following properties:

   (i) $T_{u,u} = Id$;

   (ii) $T_{v,u} = T_{u,v}^{-1}$;

   (iii) a vector $T_{u,v}(x)$ depends continuously on $u, v, x$;

   (iv) $T_{u,v}(x) = x \pmod{\mathbb{R}^j}$ if $u, v \in \mathbb{R}^j$.

(13) Do Exercise 4.12 from the notes.

(14) Do Exercise 4.13 from the notes.

(15*) Do Exercise 4.14 from the notes.