FINAL EXAM REVIEW-I

(1) Find eigenvalues and eigenvectors of the following matrices:

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(2) Compute the matrix e^{At} if

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} \lambda & 0 & 0 \\ 1 & \lambda & 0 \\ 0 & 1 & \lambda \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

(3) Compute the matrices A^k and e^A for the matrix $A = \begin{bmatrix} 0 & \beta \\ -\beta & 0 \end{bmatrix}$.

(4) Consider the system:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k & -b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

where $b \geq 0$ and $k > 0$.

- **(a)** For which values of b and k this system has complex eigenvalues? Repeated eigenvalues?
- **(b)** Find the general solution of this system in each case.

(5) Find the general solution of the system:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -2 \\ 0 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Sketch few trajectories.

(6) For small $\epsilon > 0$, find general solution of the system:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \epsilon & 1 & 0 \\ -1 & \epsilon & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Sketch few trajectories.

(7) For small $\epsilon > 0$, find the general solution of the system:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \alpha & 1 & 0 \\ 0 & \alpha & 1 \\ 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Sketch few trajectories.

(8) Find the general solution of the system:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

Sketch few trajectories.

(9) Find the general solution of the system:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 5\cos t \\ 0 \end{bmatrix}.$$

(10) Find and classify the critical points of the system

$$\begin{cases} x' & = & x - y \\ y' & = & x^2 - 1 \end{cases}$$

Sketch the phase diagrams near each critical point.

(11) Find and classify the critical points of the system

$$\begin{cases} x' & = & x - y \\ y' & = & x + y - 2xy \end{cases}$$

Sketch the phase diagrams near each critical point.

(12) Consider the dynamical system:

$$\begin{cases} x' & = & -\frac{1}{2}x^3 + 2xy^2 \\ y' & = & -y^3 \end{cases}$$

The point $(0, 0)$ is a critical one for the system. Find a Lypunov function to show that this point is asymptotically stable.

(13) Consider the dynamical system:

$$\begin{cases} x' & = & 2y(z-1) \\ y' & = & -x(z-1) \\ z' & = & -z^3 \end{cases}$$

The point $(0, 0, 0)$ is a critical one for the system. Find a Lypunov function to show that this point is asymptotically stable.