

SECOND MIDTERM REVIEW PROBLEMS

1. Evaluate the integrals using the integration by parts:

(a) $\int_1^{\sqrt{3}} \arctan(1/x) dx$

(b) $\int_1^2 x^4 (\ln x)^2 dx$

(c) $\int \sin \sqrt{x} dx$

2. Evaluate the integral or show that it is divergent:

(a) $\int_1^{\infty} \frac{1}{(2x+3)^3} dx$

(b) $\int_0^{\infty} \frac{\ln x}{x^4} dx$

(c) $\int_1^e \frac{dx}{x\sqrt{\ln x}}$

(d) $\int_2^6 \frac{x}{(x-2)} dx$

3. Find for which parameters a the integral $\int_0^{\infty} e^{ax} \cos x dx$ is convergent.

4. Find the area of the region enclosed by the curves

(a) $y = e^x, y = xe^x, x = 0$;

(b) $y = 1/x, y = x, y = 1/4x, x > 0$;

(c) $x = a \cos \theta, y = b \sin \theta, 0 \leq \theta \leq 2\pi$.

5. For which values of k the line $y = kx$ and the curve $y = \frac{x}{x^2+1}$ enclose a region? Find the area of the region.

6. Find the volume of the round ball of radius r

7. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

(a) $y - x^3, y = x, x \geq 0$; about the x -axis;

(b) $y = x, y = \sqrt{x}$; about $y = 1$;

(c) $y = e^x, y = 1, x = 2$; about $y = 3$;

(d) $y = 1 + \sec x, y = 3$; about $y = 1$.

8. Find a volume of a cap of a sphere with radius R and height h .

9. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y -axis. Sketch the region and a typical shell.

(a) $y = e^x, y = e^{-x}, x = 1$;

(b) $y = 4(x-2)^2, y = x^2 - 4x + 7$.

10. Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x -axis.

- (a) $x = 1 + (y - 2)^2$, $x = 2$;
 (b) $x + y = 3$, $x = 4 - (y - 1)^2$.

11. Evaluate the following indefinite integrals:

- (a) $\int \sin 7\theta \sin 9\theta \, d\theta$
 (b) $\int \tan^4 x \, dx$
 (c) $\int \sec^6 x \, dx$
 (d) $\int \frac{dx}{\cos x - 1}$
 (e) $\int (1 + \cos \theta)^2 \, d\theta$
 (f) $\int \tan^3 x \sec x \, dx$
 (g) $\int \frac{1 - \sin x}{\cos x} \, dx$
 (h) $\int \sin^5 x \, dx$

12. Use the method of partial fraction to evaluate the following integrals:

- (a) $\int \frac{x^3}{(x + 1)^3} \, dx$
 (b) $\int \frac{x^2 - x + 6}{x^3 + 3x} \, dx$
 (c) $\int \frac{1}{x^3 - 1} \, dx$

13. Evaluate the following integrals using any methods of your choice:

- (a) $\int \frac{1}{x\sqrt{4x + 1}} \, dx$
 (b) $\int e^x \sqrt{1 + e^x} \, dx$
 (c) $\int \frac{\ln(x + 1)}{x^2} \, dx$
 (d) $\int (\arcsin x)^2 \, dx$

14. A torus is generated by rotating the circle $x^2 + (y - R)^2 = r^2$ (where $R > r$) about the x -axis. Use any method to find the volume enclosed by the torus.

15. A 1600-lb elevator is suspended by a 200-ft cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance 300 ft?

16. An aquarium 2 m long, 1 m wide, and 1 m deep is full of water. Find the work needed to pump half of the water out of the aquarium. (Use the fact that the density of the water is 1000 kg/m^3 .)

17. The region bounded by the curves

$$x = 1 - y^4, \quad x = 0,$$

is rotated about the line $x = 2$. Use any method to find the volume of the resulting solid.