Math 252, Winter 2013 Boris Botvinnik

MIDTERM PRACTICE TEST

INSTRUCTIONS: Do all problems below, try to "time" your work.

(1) Evaluate the following definite integral

$$\int_0^1 (x+\sqrt{1-x^2})dx$$

by interpreting it in terms of areas. Show all details of your work.

(2) Use the definition of a definite integral in terms of Riemannian sums to evaluate the integral

$$\int_0^3 (x^2+x)dx.$$

Show all details of your work.

(3) Find a derivative $g'(x)$ of the following function

$$g(x)=\int_0^{\sqrt{x}} (1-t^2)e^{t^2}dt.$$

Show all details of your work.

(4) We recall that $\frac{1}{x^2}=(-\frac{1}{x})'$. However, the following equality is false:

$$\int_{-2}^1 \frac{1}{x^2}dx=\left(-\frac{1}{x}\right)\Big|_{x=-2}^{x=1}=-1+\frac{1}{2}=-\frac{1}{2}.$$

Explain why Fundamental Theorem of Calculus does not work here.

(5) Evaluate the integral: $\displaystyle\int_0^1 \frac{x}{x^2+1}dx$

(6) Evaluate the integral: $\displaystyle\int_0^1 (1+x)^{2013}dx$

(7) Evaluate the integral: $\displaystyle\int_1^2 x^3\ln x\ dx$

(8) Evaluate the integral: $\displaystyle\int \tan^2(5x)\ dx$

(9) Evaluate the integral: $\displaystyle\int e^{3x}\sin 7x\ dx$

(10) Evaluate the integral: $\displaystyle\int \frac{x^3}{\sqrt{1+x^2}}\ dx$

(11) Evaluate the integral: $\displaystyle\int \arctan 4x\ dx$

(12) Evaluate the integral: $\int \arcsin 8x \; dx$

(13) Evaluate the integral: $\int x^3 \ln x \; dx$

(14) Evaluate the integral: $\int x \ln(x+1) \; dx$

(15) Evaluate the integral: $\int_0^{\pi} e^{\sin x} \sin 2x \; dx$

(16) Evaluate the integral: $\int_0^{2\pi} \cos^2(3\theta) \; d\theta$

(17) Evaluate the integral: $\int \frac{5x+1}{(2x+1)(x-1)} \; dx$

(18) Evaluate the integral: $\int \frac{x^2}{(x+4)} \; dx$

(19) Evaluate the integral: $\int \frac{x}{(x+19)} \; dx$

(20) Graph the function $f(x) = \frac{x^3}{\sqrt{x^2+1}}$ for $0 \leq x \leq 2$. Find the area under the graph of $f(x)$ for $0 \leq x \leq 2$.

(21) Evaluate the integral: $\int \tan x \; dx$

(22) True-False Question: The integral $\int_0^2 (x-x^2) \; dx$ represents the area under the curve $y = x-x^2$ for $0 \leq x \leq 2$.

(23) True-False Question: $\int_0^4 \frac{x}{x^2-1} \; dx = \frac{\ln 15}{2}$.

(24) True-False Question: For any function $f(x)$ defined on $[a,b]$ there exists an antiderivative $F(x)$: $F'(x) = f(x)$.

(25) True-False Question: If $g(x)$ is a function on $[a,b]$ such that $g'(x)$ is continuous on $[a,b]$, then

$$\int_a^b g'(x) \; dx = g(b) - g(a).$$

(26) Use the substitution $x = \sec\theta$ to evaluate the integral: $\int_1^{\sqrt{2}} \frac{\sqrt{x^2-1}}{x^4} \; dx$

(27) Evaluate the integral: $\int_{\sqrt{2}}^2 \frac{1}{x^3\sqrt{x^2-1}} \; dx$

(28) Evaluate the integral: $\int_{16}^{25} \frac{\sqrt{x}}{x-1} \; dx$