REVIEW FOR THE MIDTERM TEST I

1. Let $A \subset \Sigma^*$ be a language, $\Sigma = \{0, 1\}$. Provide a recursive definitions for the following languages:

- **(i)** $x \in A$ if and only if x has odd number of 0's;
- **(ii)** $x \in A$ if and only if x has even number of 1's;
- **(iii)** $x \in A$ if and only if x has odd number of 0's and even number of 1's;
- **(iv)** $x \in A$ if and only if x has odd number of 0's or even number of 1's.

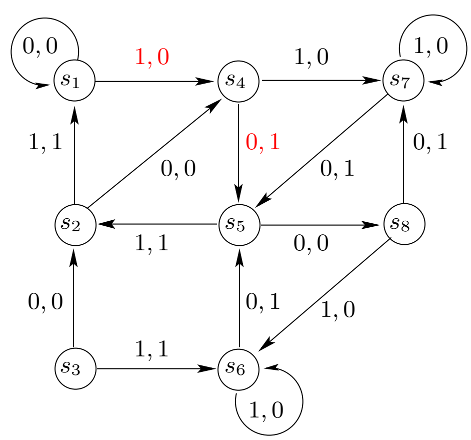
2. Let S be a set of integers $S = \{ n \mid 1 \leq n \leq 10,000 \}$.

- **(i)** How many numbers in S are divisible by 3?
- **(ii)** How many numbers in S are divisible by 7?
- **(iii)** How many numbers in S are divisible by 3 and by 7?
- **(iv)** How many numbers in S are divisible by 3 or by 7?

3. Design a finite state machine $M = (S, O, \nu, \omega)$, where $S = O = \{0, 1\}$, which recognizes a patern "1101" in a binary string.

4. Design a finite state machine $M = (S, O, \nu, \omega)$, where $S = O = \{0, 1\}$, which recognizes a patern "1101" in a binary string only when the zero occurs at the position which is multiple of 3.

5. Consider the finite state machine $M = (S, O, \nu, \omega)$, where $S = O = \{0, 1\}$, given by the diagram:



- **(i)** Write the output of the string 001100110011.
- **(ii)** Write the transitional table for the machine.
- **(iii)** Apply the minimization process to this machine.

6. Let $A = \{1, 2, 3, 4, 5\}$.

- **(i)** Determine the number of reflexive relations on A.
- **(ii)** Determine the number of symmetric relations on A.
- **(iii)** Determine the number of reflexive and symmetric relations on A.
- **(iv)** Determine the number of antisymmetric relations on A.
- **(v)** Determine the number of reflexive and antisymmetric relations on A.

7. Let A be a set of all divisors of 180. Find the number of pairs (a, b) such that a divides b, and $a, b \in A$.

8. These questions about placing identical objects to distinguished boxes.

(i) How many ways to put 14 objects to 3 boxes?

(ii) How many ways to put 14 objects to 3 boxes with at least 8 object in one box?

(iii) How many ways to put 14 objects to 3 boxes with no more than 7 object in one box?

(iv) For how many numbers between 0 and 999 the sum of their digits equal to 20?

9. Let $n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ be a decomposition of an integer n through primes. Let $D(n)$ be a set off all divisors of n. For two divisors d, d' we write $d \leq d'$ (or $(d, d') \in \mathcal{R}$) iff d divides d'.

(i) How many divisors of n are there? (What is the size $|D(n)|$?)

(ii) How many pairs (d, d') such that d divides d' are there? (What is the size $|\mathcal{R}|$?)

11. Let A be a set with n elements, $\mathcal{R}$ be a relation on A, and $M = M(\mathcal{R})$ denotes the $(0, 1)$-matrix corresponding to the relation $\mathcal{R}$.

(i) Prove that $\mathcal{R}$ is reflexive if and only if $I_n \leq M$.

(ii) Prove that $\mathcal{R}$ is symmetric if and only if $M = M^T$.

(iii) Prove that $\mathcal{R}$ is transitive if and only if $M^2 \leq M$.

12. Let A be a finite poset. Prove that A has a maximal element.

13. Let $X = \{0, 1\}$ and $A = X \times X$. We define a partial order on A:
$(a, b) \leq (b, c)$ if and only if
- $a < c$ or
- $a = c$ and $b \leq d$.

(i) Determine all maximal and minimal elements for this partial order.

(ii) Is there a least element?

(iii) Draw the Hasse diagram of this partial order.

14. Let $A = \{a_1, \ldots, a_m\}$ be a set. Find a formula to count all partitions of A. Find the number of partitions of A for $m = 4, 5$.

15. Prove that there is one-to-one correspondence between a set of equivalence relations on A and a set of partitions of A.

16. Let A and B be finite sets with $|A| = 5$, $|B| = 3$. How many onto functions $f : A \to B$ are there?

17. Let $A = \{a_1, a_2, a_3, a_4\}$. Determine the number of equivalence relations on A.

18. Let two integers m, m' be equivalent if $m - m'$ is divisible by 9. Denote by $\mathbf{Z}/9 = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ be the set of equivalent classes. Describe the multiplication table on $\mathbf{Z}/9$. In particular,

- find $x \in \mathbf{Z}/9$ such that $7 \cdot x = 1$ in $\mathbf{Z}/9$;
- find all $x \in \mathbf{Z}/9$ such that $x^2 = 1$ in $\mathbf{Z}/9$;
- find all $y \in \mathbf{Z}/9$ such that there exists $z \neq 0$ that $y \cdot z = 0$ in $\mathbf{Z}/9$.