

Summary on Lecture 22, May 23rd, 2016

Turing Machines: More Examples.

Example 4. Let us design a Turing Machine to compute the function $f(n) = (n, n)$ for a nonnegative integer n , i.e., we need a machine which performs as follows:

$$0 \underbrace{\downarrow 11 \dots 11}_n 0 \mapsto 0 \underbrace{\downarrow 11 \dots 11}_n 0 \underbrace{11 \dots 11}_n$$

We will do this in two steps.

Step 1. Here we will design a Turing Machine which does the following:

$$0 \underbrace{\downarrow 11 \dots 11}_n 0 \mapsto 0 \underbrace{11 \dots 11}_n 0 \underbrace{\downarrow 11 \dots 11}_n$$

It is difficult to do directly. We'll split this up further by describing a Turing Machine which does the following:

$$0 \underbrace{11 \dots 1}_r \underbrace{\downarrow 11 \dots 1}_{n-r} 0 \underbrace{11 \dots 1}_r \mapsto 0 \underbrace{11 \dots 1}_{r+1} \underbrace{\downarrow 11 \dots 1}_{n-r-1} 0 \underbrace{11 \dots 1}_{r+1}$$

Here is the table describing the machine:

	0	1
0		0R1
1	0R2	1R1
2	1L3	1R2
3	0L4	1L3
4	1R0	1L4

We check the sequence of actions when we start with $0 \downarrow 111010$:

$$\begin{aligned} & 0 \downarrow 111010 \xrightarrow{0R1} 010 \downarrow 11010 \xrightarrow{1R1} 0101 \downarrow 1010 \xrightarrow{1R1} 01011 \downarrow 010 \xrightarrow{0R2} 010110 \downarrow 10 \xrightarrow{1R2} 0101101 \downarrow 0 \xrightarrow{1L3} 01011011 \downarrow 1 \xrightarrow{1L3} \\ & 01011011 \downarrow 1 \xrightarrow{3} 01011011 \downarrow 1 \xrightarrow{4} 01011011 \downarrow 1 \xrightarrow{4} 01011011 \downarrow 1 \xrightarrow{4} 01011011 \downarrow 1 \xrightarrow{1R0} 0111 \downarrow 11011 \end{aligned}$$

We notice that we have a loop, and the machine does not have an assignment at the situation

$$0 \underbrace{\downarrow 11 \dots 11}_n 0 \underbrace{11 \dots 11}_n$$

Step 2. Now we describe the machine which will finish the work:

$$0 \underbrace{11 \dots 11}_n 0 \underbrace{\downarrow 11 \dots 11}_n \mapsto 0 \underbrace{\downarrow 11 \dots 11}_n 0 \underbrace{11 \dots 11}_n$$

Here is the table describing this machine:

	0	1
0	0L5	
5	0R6	1L5

We check the actions if we start with $011110 \downarrow 11110$:

$$\begin{aligned} & 011110 \downarrow 11110 \xrightarrow{0L5} 011110 \downarrow 11110 \xrightarrow{5} 011110 \downarrow 11110 \xrightarrow{5} 011110 \downarrow 11110 \xrightarrow{5} 011110 \downarrow 11110 \xrightarrow{5} 011110 \downarrow 11110 \xrightarrow{5} \\ & 011110 \downarrow 11110 \xrightarrow{5} 011110 \downarrow 11110 \xrightarrow{6} 011110 \downarrow 11110 \end{aligned}$$

Now we can combine Steps 1 and 2 to give the following design:

	0	1
0	0L5	0R1
1	0R2	1R1
2	1L3	1R2
3	0L4	1L3
4	1R0	1L4
5	0R6	1L5

Example 5. Now we would like to build a Turing Machine to compute the function $f(n) = 2n$. We will combine two designs: first, we compute the function $f_1(n) = (n, n)$ which is described in Example 4, and then we use the design of the Turing machine computing the sum $f_2(n, n) = n + n$. Here are two designs of the combination:

	0	1
0	0L5	0R1
1	0R2	1R1
2	1L3	1R2
3	0L4	1L3
4	1R0	1L4
5	0R6	1L5
6		0R7
7	1L8	1R7
8	0R9	1L8

	0	1
0	0L5	0R1
1	0R2	1R1
2	1L3	1R2
3	0L4	1L3
4	1R0	1L4
5	0R6	1L5
6	1L7	1R6
7	0R8	1L7
8		0R9

Exercise 1. Check the instructions to compute $2 \cdot 4$.

Exercise 2. For given integer $q \geq 1$, design a Turing Machine computing the function $f(n) = q \cdot n$, where $n \geq 1$.