REVIEW FOR THE MIDTERM TEST I

1. Let $A \subset \Sigma^*$ be a language, $\Sigma = \{0,1\}$. Provide a recursive definitions for the following languages:

(i) $x \in A$ if and only if x has odd number of 0's;

(ii) $x \in A$ if and only if x has even number of 1's;

(iii) $x \in A$ if and only if x has odd number of 0's and even number of 1's;

(iv) $x \in A$ if and only if x has odd number of 0's or even number of 1's.

2. Let S be a set of integers $S = \{ \ n \mid 1 \leq n \leq 10,000 \ \}$.

(i) How many numbers in S are divisible by 3?

(ii) How many numbers in S are divisible by 7?

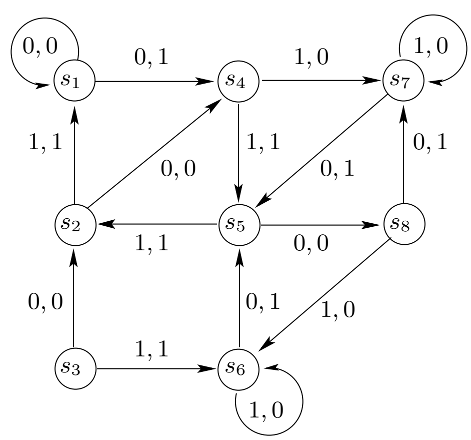
(iii) How many numbers in S are divisible by 3 and by 7?

(iv) How many numbers in S are divisible by 3 or by 7?

3. Design a finite state machine $M = (S, O, \nu, \omega)$, where $S = O = \{0,1\}$, which recognized a patern "1101" in a binary string.

4. Design a finite state machine $M = (S, O, \nu, \omega)$, where $S = O = \{0,1\}$, which recognized a patern "1101" in a binary string only when the zero occurs at the position which is multiple of 3.

5. Consider the finite state machine $M = (S, O, \nu, \omega)$, where $S = O = \{0,1\}$, given by the diagram:



(i) Write the output of the string 001100110011.

(ii) Write the transitional table for the machine.

(iii) Apply the minimization process to this machine.

6. Let $A = \{1,2,3,4,5\}$.

(i) Determine the number of reflexive relations on A.

(ii) Determine the number of symmetric relations on A.

(iii) Determine the number of reflexive and symmetric relations on A.

(iv) Determine the number of antisymmetric relations on A.

(v) Determine the number of reflexive and antisymmetric relations on A.

7. Let A be a set of all divisors of 180. Find the number of pairs (a, b) such that a divides b, and $a, b \in A$.

8. These questions about placing identical objects to distinguished boxes.

(i) How many ways to put 14 objects to 3 boxes?

(ii) How many ways to put 14 objects to 3 boxes with at least 8 object in one box?

(iii) How many ways to put 14 objects to 3 boxes with no more than 7 object in one box?

(iv) For how many numbers between 0 and 999 the sum of their digits equal to 20?

9. Let $n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ be a decomposition of an integer n through primes. Let $D(n)$ be a set off all divisors of n. For two divisors d, d' we write $d \leq d'$ (or $(d, d') \in \mathcal{R}$) iff d divides d'.

(i) How many divisors of n are there? (What is the size $|D(n)|$?)

(ii) How many pairs (d, d') such that d divides d' are there? (What is the size $|\mathcal{R}|$?)

11. Let A be a set with n elements, $\mathcal{R}$ be a relation on A, and $M = M(\mathcal{R})$ denotes the $(0, 1)$-matrix corresponding to the relation $\mathcal{R}$.

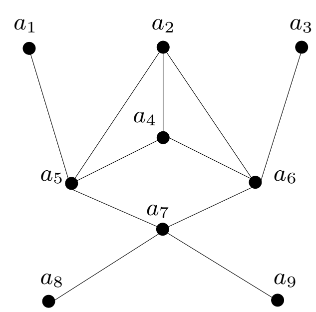
(i) Prove that $\mathcal{R}$ is reflexive if and only if $I_n \leq M$.

(ii) Prove that $\mathcal{R}$ is symmetric if and only if $M = M^T$.

(iii) Prove that $\mathcal{R}$ is transitive if and only if $M^2 \leq M$.

12. Let A be a finite poset. Prove that A has a maximal element.

13. For a poset $A = \{a_1, \dots, a_9\}$, the Hasse diagram is shown below. Topologically sort this Hasse diagram.



14. Let $X = \{0, 1\}$ and $A = X \times X$. We define a partial order on A:

$(a, b) \leq (b, c)$ if and only if

- $a < c$ or
- $a = c$ and $b \leq d$.

(i) Determine all maximal and minimal elements for this partial order.

(ii) Is there a least element?

(iii) Prove that $\mathcal{R}$ is transitive if and only if $M^2 \leq M$.

(iv) Draw the Hasse diagram of this partial order.

15. Let $A = \{a_1, \dots, a_m\}$ be a set. Find a formula to count all partitions of A. Find the number of partitions of A for $m = 4, 5, 6$.

16. Let A and B be finite sets with $|A| = 5$, $|B| = 3$. How many onto functions $f : A \to B$ are there?

17. Prove that $\mathbf{Z}/n$ is a field iff n is a prime.

18. Find 304^{-1} in $\mathbf{Z}/2015$.

19. Find 33^{-1} in $\mathbf{Z}/500$.

20. Let $A = \{a_1, a_2, a_3, a_4\}$. Determine the number of equivalence relations on A.