Homework, due to 12:00 pm, May 1, 2015

(1) Solve the system of congruences:
   (a) \[
   \begin{cases}
   x \equiv 3 \mod 7 \\
   x \equiv 4 \mod 9
   \end{cases}
   \]
   (b) \[
   \begin{cases}
   x \equiv 137 \mod 423 \\
   x \equiv 87 \mod 191
   \end{cases}
   \]
   (c) \[
   \begin{cases}
   x \equiv 37 \mod 43 \\
   x \equiv 22 \mod 49 \\
   x \equiv 18 \mod 71
   \end{cases}
   \]

(2) Do the following modular computations.
   (a) \[347 + 513 \equiv ??? \mod 763.\]
   (b) \[3274 + 1238 + 7231 + 6437 \equiv ??? \mod 9254.\]
   (c) \[153 \cdot 287 \equiv ??? \mod 353.\]
   (d) \[357 \cdot 862 \cdot 193 \equiv ??? \mod 943.\]
   (e) \[5327 \cdot 6135 \cdot 7139 \cdot 2187 \cdot 5219 \cdot 1873 \equiv ??? \mod 8157.\]
      (Hint. After each multiplication, reduce modulo 8157 before doing the next multiplication.)
   (f) \[137^2 \equiv ??? \mod 327.\]
   (g) \[373^6 \equiv ??? \mod 581.\]
   (h) \[23^3 \cdot 19^5 \cdot 11^4 \equiv ??? \mod 97.\]

(3) Find all values of \(x\) between 0 and \(m - 1\) that are solutions of the following congruences.
   (Hint. If you can’t figure out a clever way to find the solution(s), you can just substitute each value \(x = 1, 2, \ldots, m - 1\) and see which ones work.)
   (a) \[x + 17 \equiv 23 \mod 37.\]
   (b) \[x + 42 \equiv 19 \mod 51.\]
   (c) \[x^2 \equiv 3 \mod 11.\]
   (d) \[x^2 \equiv 2 \mod 13.\]
   (e) \[x^2 \equiv 1 \mod 8.\]
   (f) \[x^3 - x^2 + 2x - 2 \equiv 0 \mod 11.\]
   (g) \[x \equiv 1 \mod 5\] and also \(x \equiv 2 \mod 7\). (Find all solutions modulo 35)

(4) Suppose that \(k^a \equiv 1 \mod m\) and that \(k^b \equiv 1 \mod m\). Prove that \(k^{\gcd(a,b)} \equiv 1 \mod m\).

(5) Compute \(2015^{-1} \mod 9431\) in two ways. First, by using the fast powering algorithm, and second, by using the Euclidian algorithm.

(6) Check whether 77,060,701 is a prime number by computing \(2^{77,060,700} \mod 77,060,701\).