Homework, due to 12:00 pm, May 1, 2015

(1) Solve the system of congruences:

(a) $\begin{cases} x \equiv 3 \mod 7 \\ x \equiv 4 \mod 9 \end{cases}$

(b) $\begin{cases} x \equiv 137 \mod 423 \\ x \equiv 87 \mod 191 \end{cases}$

(c) $\begin{cases} x \equiv 37 \mod 43 \\ x \equiv 22 \mod 49 \\ x \equiv 18 \mod 71 \end{cases}$

(2) Do the following modular computations.

(a) $347 + 513 \equiv ???$ (mod 763).

(b) $3274 + 1238 + 7231 + 6437 \equiv ???$ (mod 9254).

(c) $153 \cdot 287 \equiv ???$ (mod 353).

(d) $357 \cdot 862 \cdot 193 \equiv ???$ (mod 943).

(e) $5327 \cdot 6135 \cdot 7139 \cdot 2187 \cdot 5219 \cdot 1873 \equiv ???$ (mod 8157).
(**Hint.** After each multiplication, reduce modulo 8157 before doing the next multiplication.)

(f) $137^2 \equiv ???$ (mod 327)

(g) $373^6 \equiv ???$ (mod 581)

(h) $23^3 \cdot 19^5 \cdot 11^4 \equiv ???$ (mod 97).

(3) Find all values of x between 0 and $m-1$ that are solutions of the following congruences.
(**Hint.** If you cant figure out a clever way to find the solution(s), you can just substitute each value $x = 1, 2, \ldots, m-1$ and see which ones work.)

(a) $x + 17 \equiv 23 \pmod{37}$.

(b) $x + 42 \equiv 19 \pmod{51}$.

(c) $x^2 \equiv 3 \pmod{11}$.

(d) $x^2 \equiv 2 \pmod{13}$.

(e) $x^2 \equiv 1 \pmod{8}$.

(f) $x^3 - x^2 + 2x - 2 \equiv 0 \pmod{11}$.

(g) $x \equiv 1 \pmod{5}$ and also $x \equiv 2 \pmod{7}$. (Find all solutions modulo 35)

(4) Suppose that $k^a \equiv 1 \pmod{m}$ and that $k^b \equiv 1 \pmod{m}$. Prove that $k^{\gcd(a,b)} \equiv 1 \pmod{m}$.

(5) Compute 2015^{-1} mod 9431 in two ways. First, by using the fast powering algorithm, and second, by using the Euclidian algorithm.

(6) Check whether $77,060,701$ is a prime number by computing $2^{77,060,700}$ mod 77,060,701.