Summary on Lecture 2, March 31th, 2015

Languages and Finite State Machines

First examples. (1) The first example is a device known as **turnstile**: in order to pass through the gate one has to put a quarter coin, then the turnstile unlocks and one can open the gate by pushing a bar. There are two states here: $S = \{s_0, s_1\}$, where s_0 is "the gate is locked" and s_1 is "the gate is unlocked". The are two inputs: "insert coin" and "push". We denote the inputs as $I = \{c, p\}$. There are two outputs: "Locked" and "Unlocked". We denote the outputs as $O = \{L, U\}$. Here is the diagram describing a design of this device:

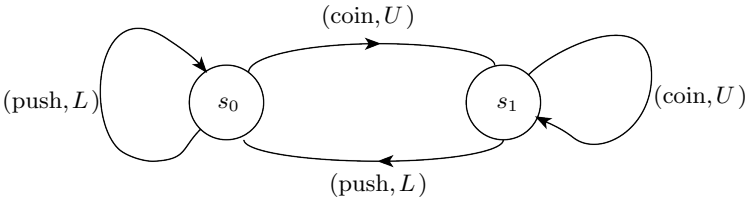

Fig. 1. Turnstile machine

Here our finite state machine $M = (S, I, O, \nu, \omega)$, where S, I, O are as above, and $\nu : S \times I \to S$ and $\omega : S \times I \to O$ are two functions describing the machine, where ν is the *next state function* and ω is the *output function*. Here we have:

$$\nu(s_0, c) = s_1, \qquad \nu(s_0, p) = s_0, \qquad \nu(s_1, c) = s_1, \qquad \nu(s_1, p) = s_0$$

$$\omega(s_0, c) = U, \qquad \nu(s_0, p) = L, \qquad \nu(s_1, c) = U, \qquad \nu(s_1, p) = U$$

The action of ν is given by arrows in the diagram, and the label over an arrow is a pair "input" and "output".

(2) Next we discuss (briefly)[1] an easy vending machine which sells two types of products **P** and **S**. The cost of each product is 20 cents. The machine accepts nickels, dimes and quarters and return necessary change.

The machine has 5 states s_0, s_1, s_2, s_3, s_4 which correspond to how much money were inserted. Namely, the state s_k corresponds to k¢, $k = 0, 1, 2, 3, 4$. Then the machine has two buttons B_P and B_S which correspond to the products **P** and **S**. Thus the set of **inputs** I we can describe as the set $I = \{5, 10, 25, B_P, B_S\}$. For each input (i.e. inserting nickels, dimes and quarters or pushing the buttons B_P or B_S), we have to describe the next state of the machine and a relevant output. The set of outputs $O = \{n, 5, 10, 15, 20, 25, P, S\}$, where n means **nothing**, the numbers 5,10,15,20,25 mean the change given back to a customer and the letters P, S mean releasing the corresponding product **P** or **S** to the customer. Below is the table describing the functions

$$\nu : S \times I \to S \quad \text{and} \quad \omega : S \times I \to O$$

	ν					ω				
	5¢	10¢	25¢	B_P	B_S	5¢	10¢	25¢	B_P	B_S
s_0	s_1	s_2	s_4	s_0	s_0	n	n	5¢	n	n
s_1	s_2	s_3	s_4	s_1	s_1	n	n	10¢	n	n
s_2	s_3	s_4	s_4	s_2	s_2	n	n	15¢	n	n
s_3	s_4	s_4	s_4	s_3	s_3	n	n	20¢	n	n
s_4	s_4	s_4	s_4	s_0	s_0	5¢	10¢	25¢	P	S

Fig. 2 below gives a diagram describing this vending machine.

[1]The textbook discusses this example in more detail.

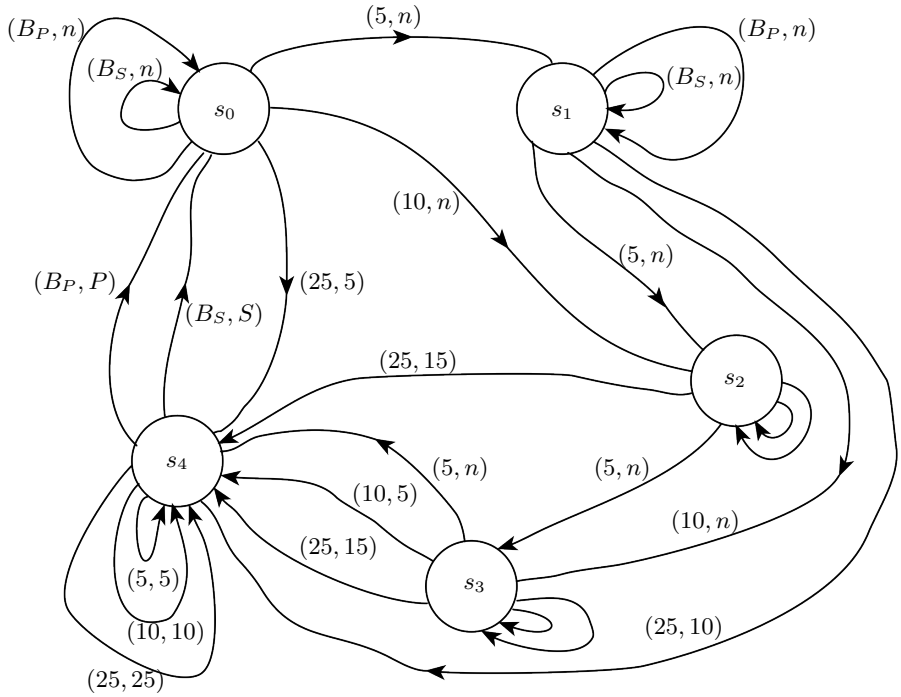

Fig. 2. Vending machine