Summary on Lecture 20, March 6, 2017

Dijkstra's Shortest Path Algorithm

Let $G = (V, E)$ be a digraph and $\mathsf{wt} : E \to (0, \infty)$ be a weighted function. We give a pseudo-code for the Dijkstra's Shortest Path Algorithm.

Dijkstra (G, v, wt)

Input: A digraph $G = (V, E)$, a vertex $v \in V$, $|V| = n+1$, a weighted function $\mathsf{wt} : E \to (0, \infty)$.
Output: A set of vertices $S = \{v_0, v_1, \ldots, v_n\}$ and a set of labels $\mathbf{L} = \{(L(v_0), u_0), (L(v_1), u_1), \ldots, (L(v_n), u_n)\}$, where $L(v_i) = d(v_0, v_i)$ and u_i is the pointer to the previous vertex of the shortest path from v_0 to v_i.

Set $j := 0$ and $S_j := \{v_0\}$, $\bar{S}_0 := V \setminus S_j$, $(L(v_0), u_0) := (0, -)$, $(L(v_i), u_i) := (\infty, -)$ for $i = j+1, \ldots, n$.

If $n = 1$, return S_j, $\mathbf{L} = \{(0, -)\}$.

Main loop: While $j < n$ do

for each $\bar{v} \in \bar{S}_j$ do

find $u \in S_j$ such that $L(u) + \mathsf{wt}(u, \bar{v})$ is minimal over all $u \in S_j$,

$L(\bar{v}) := \min\{L(\bar{v}), L(u) + \mathsf{wt}(u, \bar{v})\}$, and set the label $(L(\bar{v}), u)$ for the vertex $\bar{v}$.

If $L(\bar{v}) = \infty$ for all $\bar{v} \in \bar{S}$, return S, $\mathbf{L} := \mathbf{L} \cup \{(\infty, -), \ldots, (\infty, -)\}$ (where we add $n - j + 1$ terms).

Else find $\bar{v}_* \in \bar{S}_j$ such that $L(\bar{v}_*)$ is minimal.

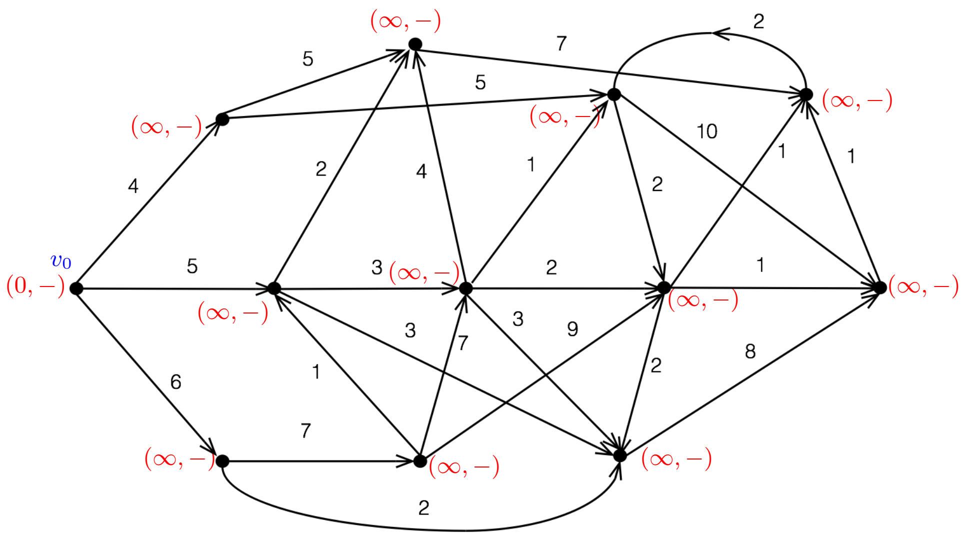
Set $v_{j+1} := \bar{v}_*$, $u_{j+1} := u$, $\mathbf{L} := \mathbf{L} \cup \{(L(v_{j+1}), u_{j+1})\}$.

Set $S_{j+1} := S_j \cup \{v_{j+1}\}$. Set $j := j+1$ and return to the **Main loop**.

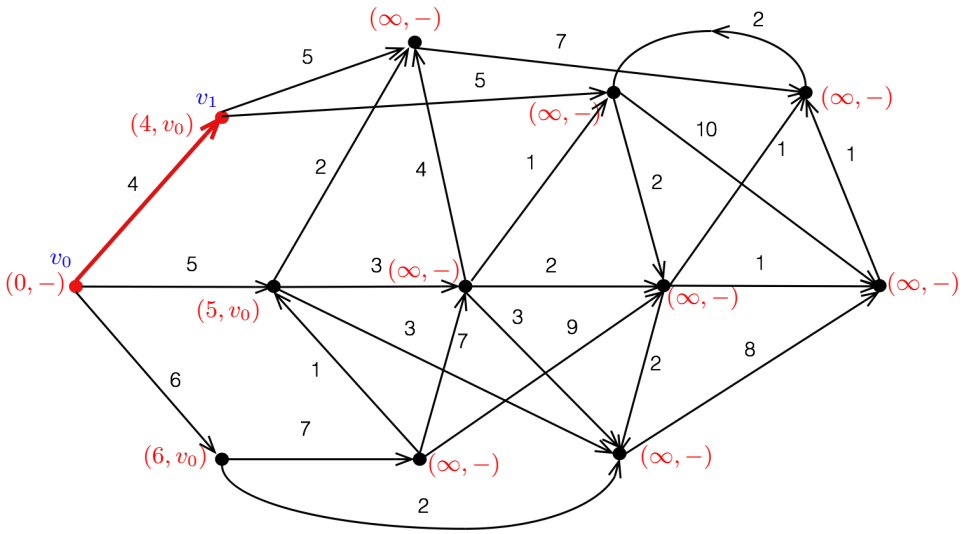
Return L

Example. Now describe the Dijkstra's Shortest Path Algorithm for the following digraph:

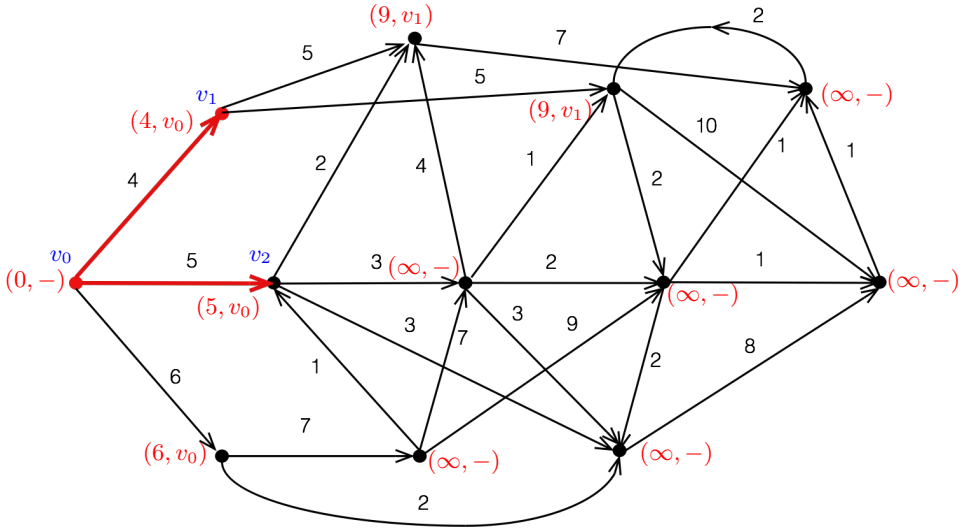
Step 1:



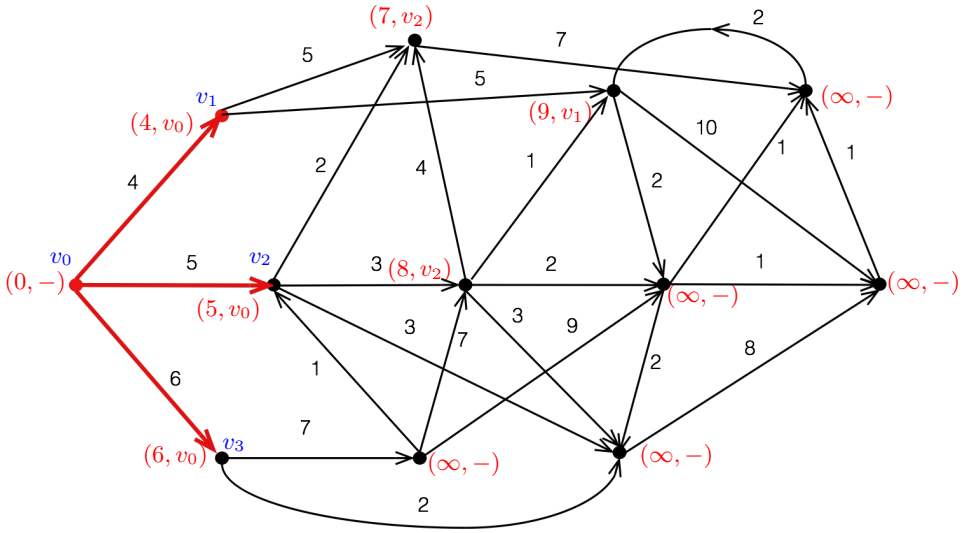
Step 2:



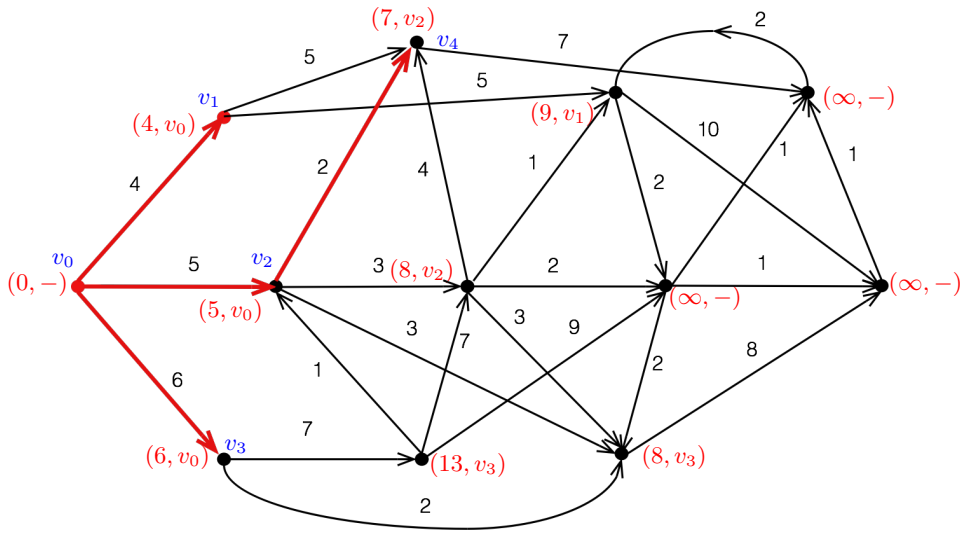
Step 3:



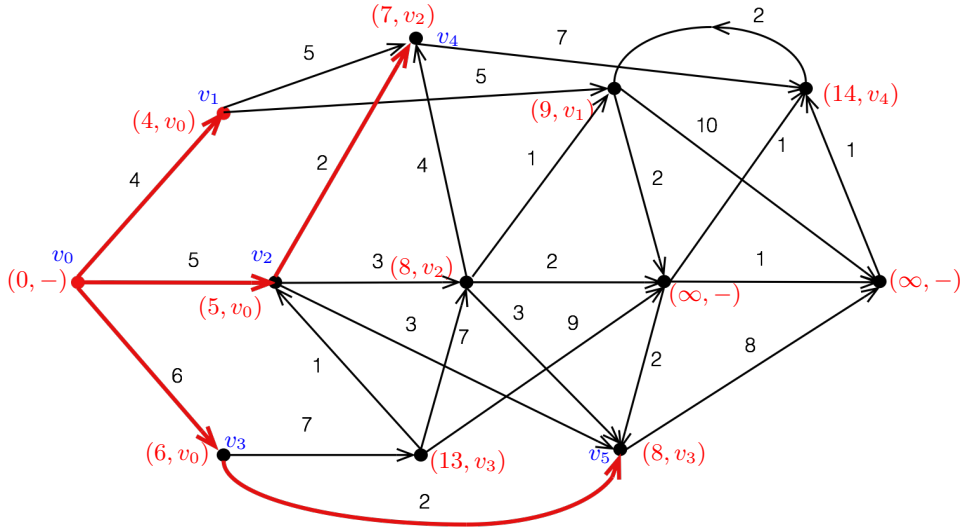
Step 4:



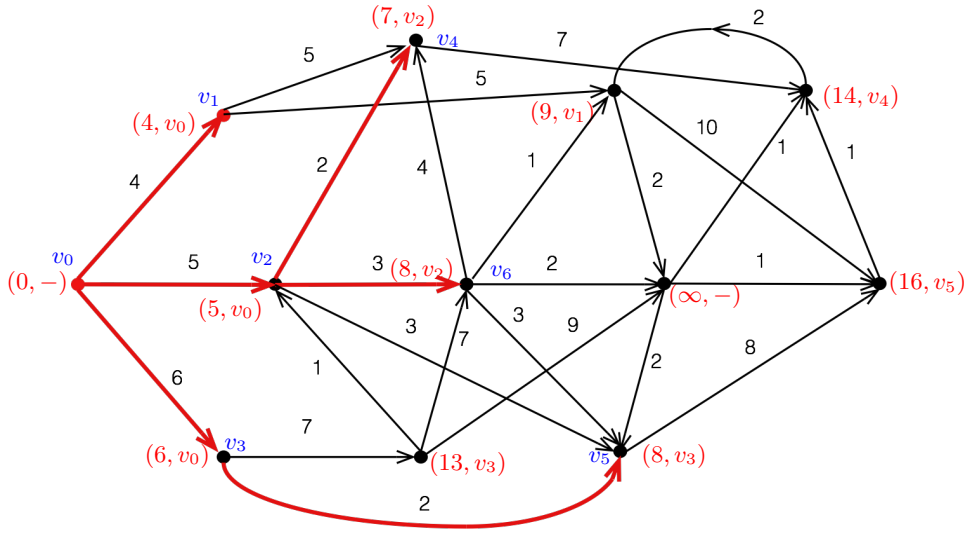
Step 5:



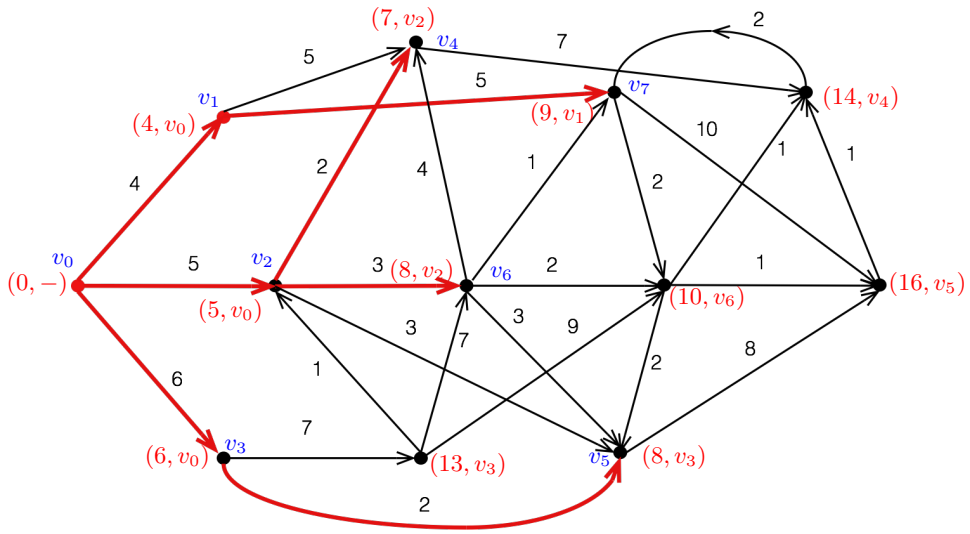
Step 6:



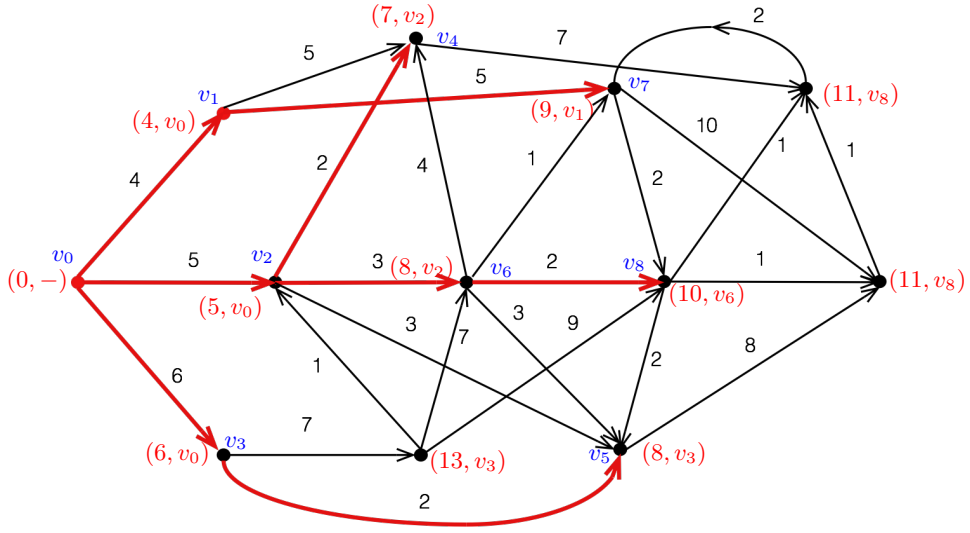
Step 7:



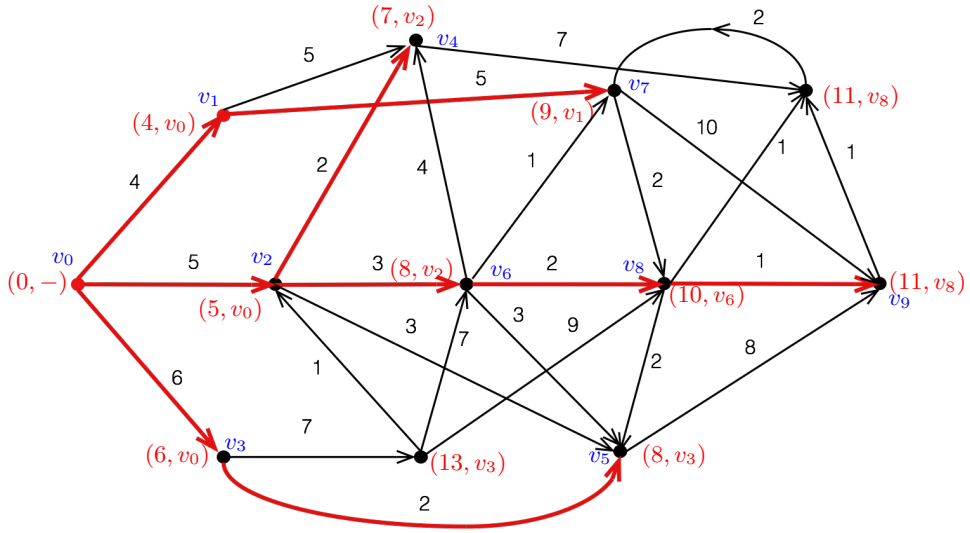
Step 8:



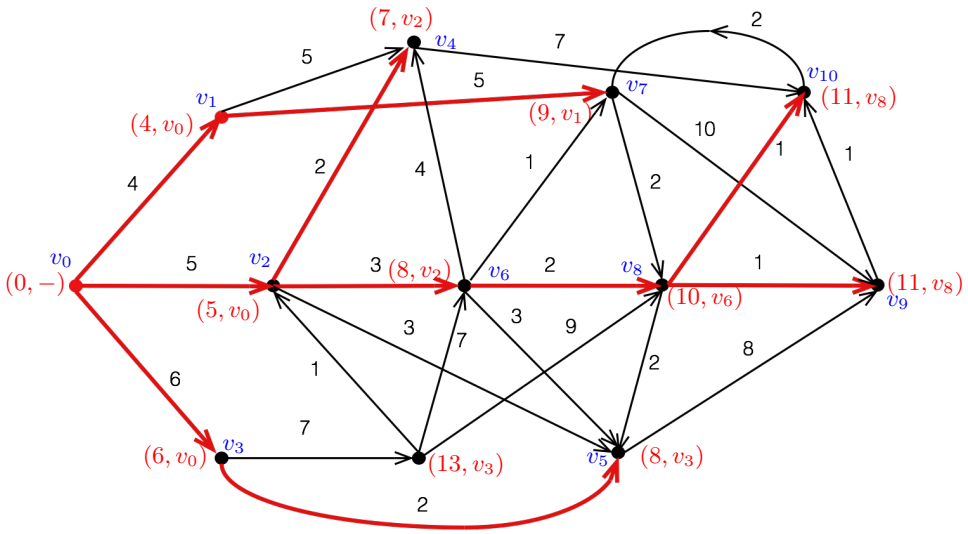
Step 9:



Step 10:



Step 11:



Step 12: