REVIEW PROBLEMS FOR THE FIRST MIDTERM TEST

1. Find an Euler circuit (if it does exist) in a given graph.

2. Let a_n be the number of words of length n in A, B, C, and D with an odd number of B's. Calculate a_0, a_1, a_2, a_3, a_4. Find a recurrence relation satisfied by a_n for all $n \geq 2$.

3. Solve the following recurrence relations:

(a) $a_n = a_{n-1} + 2a_{n-2}$, $n \geq 2$,
$a_0 = 1$, $a_1 = 1$.

(b) $a_n = a_{n-1} + a_{n-2}$, $n \geq 2$,
$a_0 = 0$, $a_1 = 1$.

(c) $a_n = 6a_{n-1} + 9a_{n-2}$, $n \geq 2$,
$a_0 = 1$, $a_1 = -3$.

(d) $a_n = 2a_{n-1} - 2a_{n-2}$, $n \geq 2$,
$a_0 = 0$, $a_1 = 1$.

4. Compute $(1 + i\sqrt{3})^{2018}$, $(1 - i)^{2018}$.

5. Use generating functions to solve the following recurrence relations:

(a) $a_n - 3a_{n-1} = 7n$, $n \geq 1$,
$a_0 = 1$.

(b) $a_n + a_{n-1} = 3n - 5n^2$, $n \geq 1$,
$a_0 = 1$,

(c) $a_n + 3a_{n-1} - 10a_{n-2} = 3 \cdot 2^n$, $n \geq 2$,
$a_0 = 0$, $a_1 = 6$.

6. Let F_n, $n = 0, 1, \ldots$ be the Fibonacci numbers. Prove that

$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \frac{1+\sqrt{5}}{2}.$$

7. Let F_n, $n = 0, 1, \ldots$ be the Fibonacci numbers. Prove that

$$\left(\tfrac{1+\sqrt{5}}{2}\right)^{n-2} < F_n < \left(\tfrac{1+\sqrt{5}}{2}\right)^{n-1}, \quad n \geq 3.$$

8. Let $\Sigma = \{0, 1\}$ be an alphabet, and $A = \{0, 01, 11\} \subset \Sigma^*$ be a language over Σ. Find a number of strings of length n over A.

9. Let a_n be the number of words of lenght n in A, B, C, and D with an odd number of B's. Calculate a_0, a_1, a_2, a_3, a_4. Find a recurrence relation satisfied by a_n for all $n \geq 2$.

10. Let $\Sigma = \{0, 1\}$ and A_n be the set of binary strings of length n which do not contain the string 00. Find and solve a recurrence relation for $a_n = |A_n|$.

11. Let $G = (V, E)$ be a finite graph. Assume $v, v' \in V$ are two verices which are connected by some $v - v'$-walk. Prove that there exists an $x - x'$-path (i.e. a walk which does not visit any vertex more than once).

12. A graph $G = (V, E)$ with 21 edges has seven vertices of degree 1, three of degree 2, seven of degree 3 and the rest of degree 4. How many vertices does it have?

13. Write an algorithm to construct a circuit for a graph G, where all vertices of G have even degree. Explain why does it work.

14. Write an algorithm to construct an Euler circuit for a graph G, where all vertices of G have even degree. Explain why does it work.