REVIEW FOR THE FINAL TEST II:

1. Design a recursive algorithm which for each integer $n > 0$ computes such k that $7^{k-1} \leq n < 7^k$.

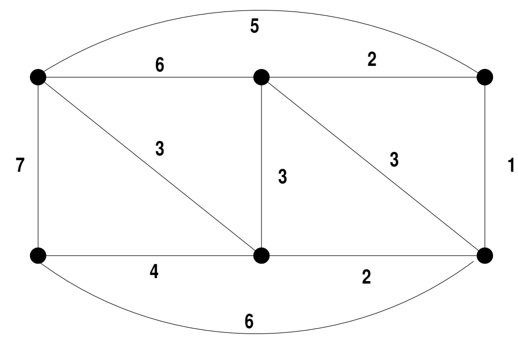
2. Let $\Sigma = \{a, b\}$, and Σ^* be the language over Σ. Describe recursively the set T of words containing at least one a's, at least one b's and where all a's precede all b's.

3. Consider the following algorithm:

CON-3$[n]$:
```
{Input:  a non-negative integer n}
{Output:  ????} ⟵ explain
if n < 3 then
   return n
else
   return CON-3[n DIV 3]n MOD 3
   {Here n MOD 3 follows the number CON-3[n DIV 3].}
```
Does the algorithm **CON-3**$[n]$ terminate?
What is the output if you evaluate **CON-3**[101]? **CON-3**[99,999]?

4. Let K_n be a complete graph with n vertices. For which n the graph K_n admits an Euler circuit? Explain in detail.

5. Write the Prim's algorithm. Use Prim's algorithm to find a minimal spanning tree for the following graph:



6. Let Q_n be a hypercube.
(a) How many vertices does Q_n have?
(b) How many edges does Q_n have?
(c) What is a degree of every vertex in Q_n?
(d) When does Q_n have an Euler circuit?

7. Find the maximum distance between pairs of vertices in Q_8.

8. How many distinct paths of length 2 are there in Q_n?

9. Compute the chromatic plynomial $P(Q_3, \lambda)$.

10. Determine the number of cyles of length 4 in the hypercube Q_n.

11. Construct an optimal tree for the following weights $\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43\}$.

12. Define a complete binary tree. How many complete binary trees are there with $2n + 1$ vertices?

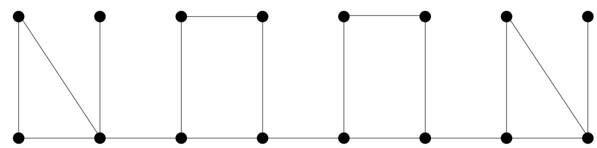
13. Let $G = (V, G)$ be a graph with no loops and parallel edges, and $|V| = n \geq 3$. Give a detailed proof that if $\deg(v) + \deg(w) \geq n$ for each pair of vertices v and w which are not connected by an edge, then G has a Hamiltonian cycle.

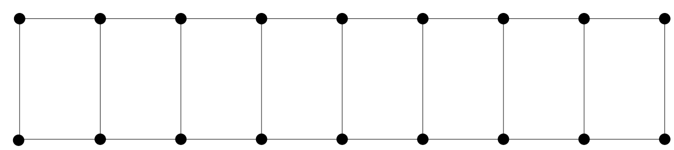
14. Write the Kruskal's algorithm. Use Kruskal's algorithm to find a minimal spanning tree for the weighted graph in problem # 5.

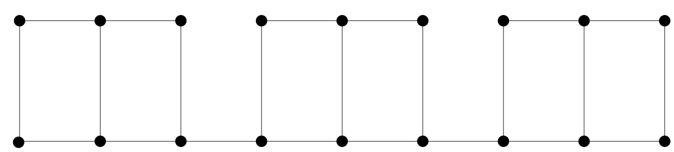
15. Define a complete binary tree. How many complete binary trees are there with $2n+1$ vertices?

16. Let $G=(V,G)$ be a graph with no loops and parallel edges, and $|V|=n\geq 3$. Give a detailed proof that if $\deg(v)+\deg(w)\geq n$ for each pair of vertices v and w which are not connected by an edge, then G has a Hamiltonian cycle.

17. Find chromatic polynomials $P(G,\lambda)$ for the following graphs:

(a) 

(b) 

(c) 

18. Prove that any tree has at least two leaves.

19. Let $T=(V,E)$ be a tree. Prove that for any two distinct vertices $v,u\in V$ there is a unique path connecting them.

20. Let $T=(V,E)$ be a tree. Prove that $|V|=|E|+1$.

21. Let T be a complete binary tree.
(a) Prove that it has odd number of vertices.
(b) Assume the height of T is h, and T has ℓ leaves. Prove that $\ell\leq 2^h$.

22. Write the expression

$$\frac{(x-1)(x^5+x^4+x^3+x^2+x+1)-(x^6-1)}{(x-1)^8}$$

in Polish notations.

23. Let $G=(V,E)$ be a finite graph.

(a) Assume that $|V|=|E|+1$ and that G is connected. Prove G is a tree.
(b) Assume that $|V|=|E|+1$. Find an example that G is not a tree.

24. A connected graph $G=(V,E)$ has 2018 edges. What is the maximal value of $|V|$? Give proof and example.

25. Let $G=(V,E)$ be a loop-free connected graph with $V=\{v_1,\ldots,v_n\}$, where $n\geq 2$, $\deg v_1=1$ and $\deg v_j\geq 2$ for all $2\leq j\leq n$. Prove that G must have a cycle.