REVIEW PROBLEMS FOR THE MIDTERM TEST

1. Find an Euler circuit (if it does exist) in a given graph.

2. Let a_n be the number of words of length n in A, B, C, and D with an odd number of B's. Calculate a_0, a_1, a_2, a_3, a_4. Find a recurrence relation satisfied by a_n for all $n \geq 2$.

3. Solve the following recurrence relations:

(a) $a_n = a_{n-1} + 2a_{n-2}$, $n \geq 2$,
$a_0 = 1$, $a_1 = 1$.

(b) $a_n = a_{n-1} + a_{n-2}$, $n \geq 2$,
$a_0 = 0$, $a_1 = 1$.

(c) $a_n = 6a_{n-1} + 9a_{n-2}$, $n \geq 2$,
$a_0 = 1$, $a_1 = -3$.

(d) $a_n = 2a_{n-1} - 2a_{n-2}$, $n \geq 2$,
$a_0 = 0$, $a_1 = 1$.

4. Use generating functions to solve the following recurrence relations:

(a) $a_n - 3a_{n-1} = n^2$, $n \geq 1$,
$a_0 = 1$.

(b) $a_n - a_{n-1} = 3n^2 - 5n^3$, $n \geq 1$,
$a_0 = 1$,

(c) $a_n + 3a_{n-1} - 10a_{n-2} = 3 \cdot 2^n$, $n \geq 2$,
$a_0 = 0$, $a_1 = 6$.

5. Let $\Sigma = \{0, 1\}$ be an alphabet, and $A = \{0, 01, 111\} \subset \Sigma^*$ be a language over Σ. Find a number of strings of length n over A.

6. Let $\Sigma = \{0, 1\}$ and A_n be the set of binary strings of length n which do not contain the string 000. Find and solve a recurrence relation for $a_n = |A_n|$.

7. A graph $G = (V, E)$ with 21 edges has seven vertices of degree 1, three of degree 2, seven of degree 3 and the rest of degree 4. How many vertices does it have?

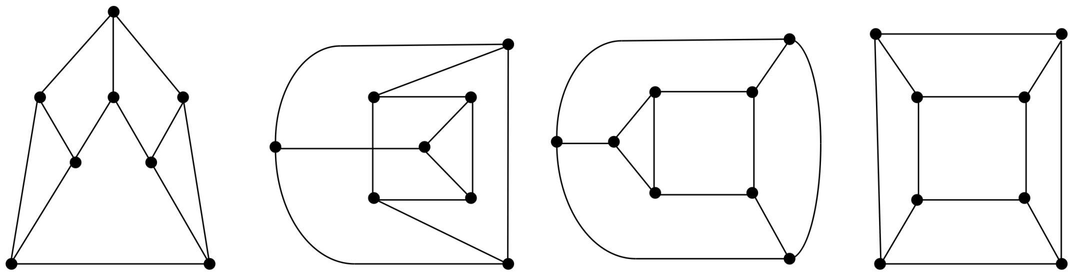
8. Prove that a connected graph G has an Euler circuit if and only if all vertices of G have even degree.

9. Let G be a connected graph with loops or multiple edges. Assume that $\deg v + \deg v' > n$ for each pair of distict vertices v and v'. Prove that G is a Hamiltonian graph.

10. Write an algorithm to construct a circuit for a graph G, where all vertices of G have even degree. Explain why does it work.

11. Write an algorithm to construct an Euler circuit for a graph G, where all vertices of G have even degree. Explain why does it work.

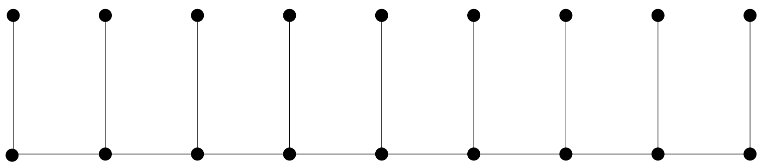
12. Which, if any, of the pairs of graphs shown are isomorphic? Justify your answer by describing an isomorphism or explaining why one does not exist.



G_1 G_2 G_3 G_4

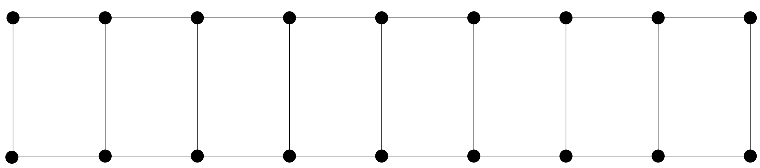
13. Compute the chromatic polynomial of the graphs G_1 and G_4.

14. Compute chromatic polynomial of the following graph G



Find $\chi(G)$.

15. Compute chromatic polynomial of the following graph G



Find $\chi(G)$.

16. Let C_n be a cycle on n vertices. Prove that $P(C_n, \lambda) = (\lambda - 1)^n + (-1)^n(\lambda - 1)$.

17. Let W_{n+1} be a wheel on $(n+1)$ vertices. Compute the chromatic polynomial of W_{n+1}. Find $\chi(W_{n+1})$.

18. Let $G = (V, G)$ be a graph. Prove that the vertices of a graph $G = (V, G)$ is twice the number of edges, i.e

$$\sum_{v \in V} \deg(v) = 2 \cdot |E(G)|.$$

19. Let $G = (V, E)$ be a graph without loops or multiple edges with $|V| = n \geq 3$. Assume that $\deg v \geq \frac{n}{2}$ for every vertex $v \in V$. Prove that G has a Hamiltonian cycle.

20. Let $G = (V, E)$ be a graph without loops or multiple edges with $|V| = n \geq 3$. Assume that G has at least $\frac{1}{2}(n-1)(n-2) + 2$ edges. Prove that G has a Hamiltonian cycle.