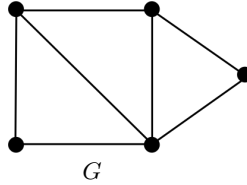


Summary on Lecture 8, July 29, 2015

More Examples on Graph Coloring and Chromatic Polynomials

**Example 1.** Find chromatic polynomial of the graph



We have that

$$P(G, \lambda) = \left( \begin{array}{c} \text{Graph with diagonal and triangle} \end{array} \right) = \left( \begin{array}{c} \text{Square with triangle} \end{array} \right) - \left( \begin{array}{c} \text{Triangle} \end{array} \right), \text{ then we decompose the first term:}$$

$$\left( \begin{array}{c} \text{Square with triangle} \end{array} \right) = \left( \begin{array}{c} \text{Square with diagonal} \end{array} \right) - \left( \begin{array}{c} \text{Square} \end{array} \right), \text{ where the first term is decomposed as:}$$

$$\left( \begin{array}{c} \text{Square with diagonal} \end{array} \right) = \left( \begin{array}{c} \text{Square} \end{array} \right) - \left( \begin{array}{c} \text{Square} \end{array} \right). \text{ Also we have:}$$

$$\left( \begin{array}{c} \text{Triangle} \end{array} \right) = \left( \begin{array}{c} \text{Triangle} \end{array} \right) - \left( \begin{array}{c} \text{Triangle} \end{array} \right) = \lambda(\lambda - 1)^3 - \lambda(\lambda - 1)^2.$$

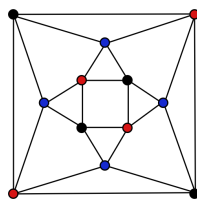
We calculate:

$$\begin{aligned} \left( \begin{array}{c} \text{Square with triangle} \end{array} \right) &= \left( \begin{array}{c} \text{Square} \end{array} \right) - 2 \left( \begin{array}{c} \text{Square} \end{array} \right) = \lambda((\lambda - 1)^4 + (\lambda - 1)) - 2((\lambda - 1)^4 + (\lambda - 1)) \\ &= ((\lambda - 1)^4 + (\lambda - 1))(\lambda - 2) \\ &= (\lambda - 1)(\lambda - 2)((\lambda - 1)^3 + 1). \end{aligned}$$

We obtain the resulting chromatic polynomial

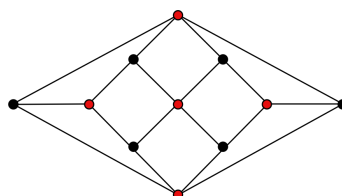
$$\begin{aligned} P(G, \lambda) &= (\lambda - 1)(\lambda - 2)((\lambda - 1)^3 + 1) - \lambda(\lambda - 1)^3 + \lambda(\lambda - 1)^2 \\ &= \lambda(\lambda - 1)(\lambda - 2)(\lambda^2 - 3\lambda + 3) - \lambda(\lambda - 1)^2(\lambda - 2) \\ &= \lambda(\lambda - 1)(\lambda - 2)(\lambda^2 - 3\lambda + 3 - \lambda + 1) \\ &= \lambda(\lambda - 1)(\lambda - 2)(\lambda^2 - 4\lambda + 4) \\ &= \lambda(\lambda - 1)(\lambda - 2)^3. \end{aligned}$$

**Exercise 1.** Find chromatic polynomial of the graph  $G_1$



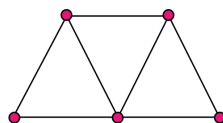
$G_1$

**Exercise 2.** Find chromatic polynomial of the graph  $G_2$



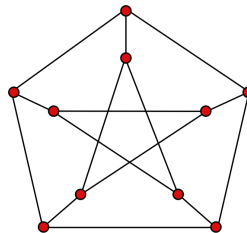
$G_2$

**Exercise 3.** Find chromatic polynomial of the graph  $G_3$



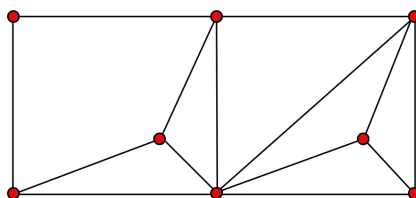
$G_3$

**Exercise 4.** Find chromatic polynomial of the graph  $G_4$



$G_4$

**Exercise 5.** Find chromatic polynomial of the graph  $G_5$



$G_5$