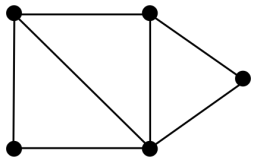
Summary on Lecture 8, July 29, 2015

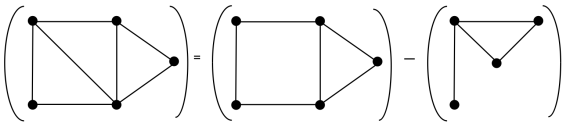
More Examples on Graph Coloring and Chromatic Polynomials

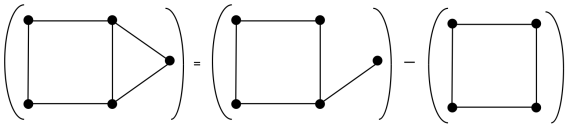
Example 1. Find chromatic polynomial of the graph

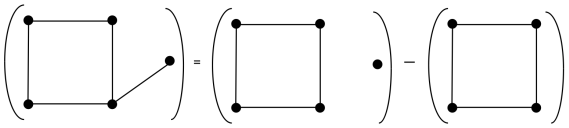


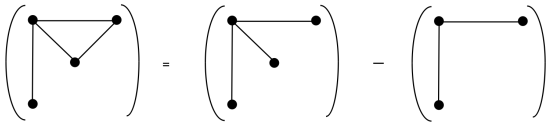
G

We have that

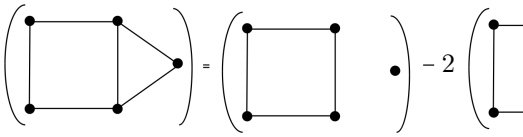
$P(G,\lambda) =$  , then we decompose the first term:

 , where the first term is decomposed as:

 . Also we have:

 $= \lambda(\lambda-1)^3 - \lambda(\lambda-1)^2$.

We calculate:



$$= \lambda((\lambda-1)^4 + (\lambda-1)) - 2((\lambda-1)^4 + (\lambda-1))$$

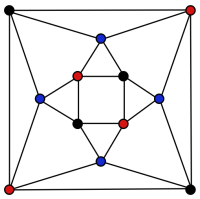
$$= ((\lambda-1)^4 + (\lambda-1))(\lambda-2)$$

$$= (\lambda-1)(\lambda-2)((\lambda-1)^3+1).$$

We obtain the resulting chromatic polinomial

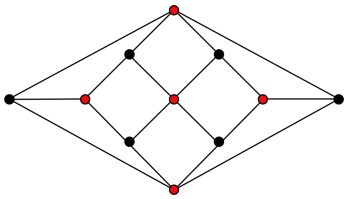
$$\begin{aligned} P(G,\lambda) &= (\lambda-1)(\lambda-2)((\lambda-1)^3+1) - \lambda(\lambda-1)^3 + \lambda(\lambda-1)^2 \\ &= \lambda(\lambda-1)(\lambda-2)(\lambda^2-3\lambda+3) - \lambda(\lambda-1)^2(\lambda-2) \\ &= \lambda(\lambda-1)(\lambda-2)(\lambda^2-3\lambda+3-\lambda+1) \\ &= \lambda(\lambda-1)(\lambda-2)(\lambda^2-4\lambda+4) \\ &= \lambda(\lambda-1)(\lambda-2)^3. \end{aligned}$$

Exercise 1. Find chromatic polynomial of the graph G_1



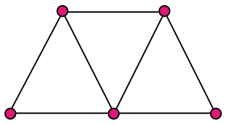
G_1

Exercise 2. Find chromatic polynomial of the graph G_2



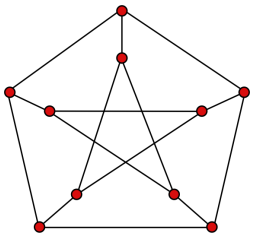
G_2

Exercise 3. Find chromatic polynomial of the graph G_3



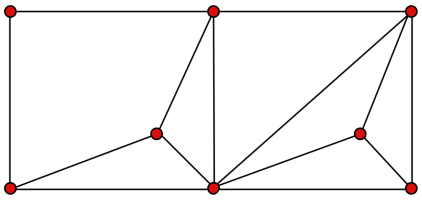
G_3

Exercise 4. Find chromatic polynomial of the graph G_4



G_4

Exercise 5. Find chromatic polynomial of the graph G_5



G_5