

## Summary on Lecture 21, November 13, 2017

## Dijkstra's Shortest Path Algorithm

Let  $G = (V, E)$  be a digraph and  $\text{wt} : E \rightarrow (0, \infty)$  be a weighted function. We give a pseudo-code for the Dijkstra's Shortest Path Algorithm.

**Dijkstra**( $G, v, \text{wt}$ )

**Input:** A digraph  $G = (V, E)$ , a vertex  $v \in V$ ,  $|V| = n + 1$ , a weighted function  $\text{wt} : E \rightarrow (0, \infty)$ .  
**Output:** A set of vertices  $S = \{v_0, v_1, \dots, v_n\}$  and a set of labels  $\mathbf{L} = \{(L(v_0), u_0), (L(v_1), u_1), \dots, (L(v_n), u_n)\}$ , where  $L(v_i) = d(v_0, v_i)$  and  $u_i$  is the pointer to the previous vertex of the shortest path from  $v_0$  to  $v_i$ .

Set  $j := 0$  and  $S_j := \{v_0\}$ ,  $\bar{S}_0 := V \setminus S_j$ ,  $(L(v_0), u_0) := (0, -)$ ,  $(L(v_i), u_i) := (\infty, -)$   
for  $i = j + 1, \dots, n$ .

If  $n = 1$ , return  $S_j$ ,  $\mathbf{L} = \{(0, -)\}$ .

**Main loop:** While  $j < n$  do

for each  $\bar{v} \in \bar{S}_j$  do

find  $u \in S_j$  such that  $L(u) + \text{wt}(u, \bar{v})$  is minimal over all  $u \in S_j$ ,

$L(\bar{v}) := \min\{L(\bar{v}), L(u) + \text{wt}(u, \bar{v})\}$ , and set the label  $(L(\bar{v}), u)$  for the vertex  $\bar{v}$ .

If  $L(\bar{v}) = \infty$  for all  $\bar{v} \in \bar{S}_j$ , return  $S$ ,  $\mathbf{L} := \mathbf{L} \cup \{(\infty, -), \dots, (\infty, -)\}$   
(where we add  $n - j + 1$  terms).

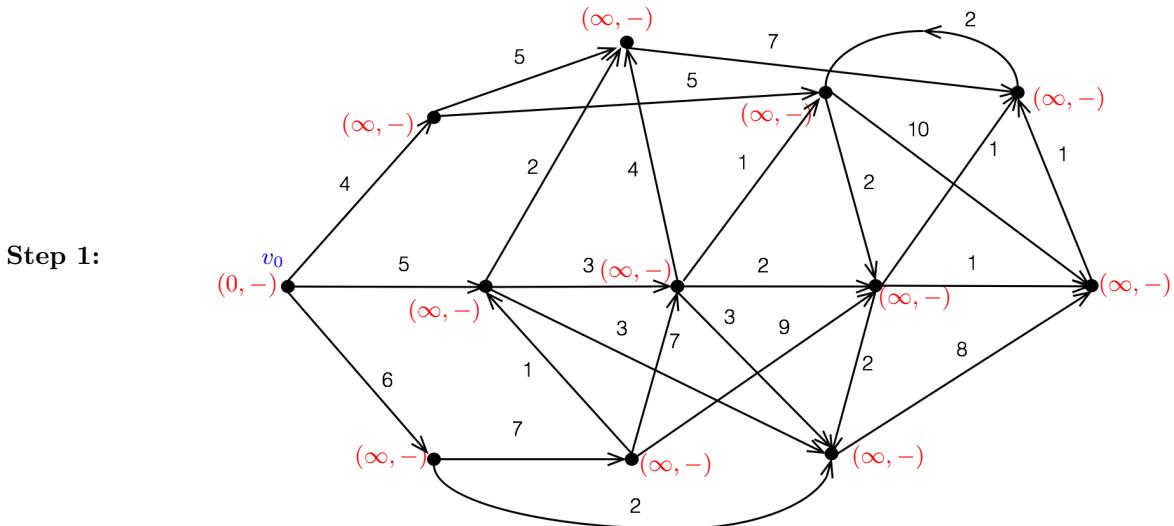
Else find  $\bar{v}_* \in \bar{S}_j$  such that  $L(\bar{v}_*)$  is minimal.

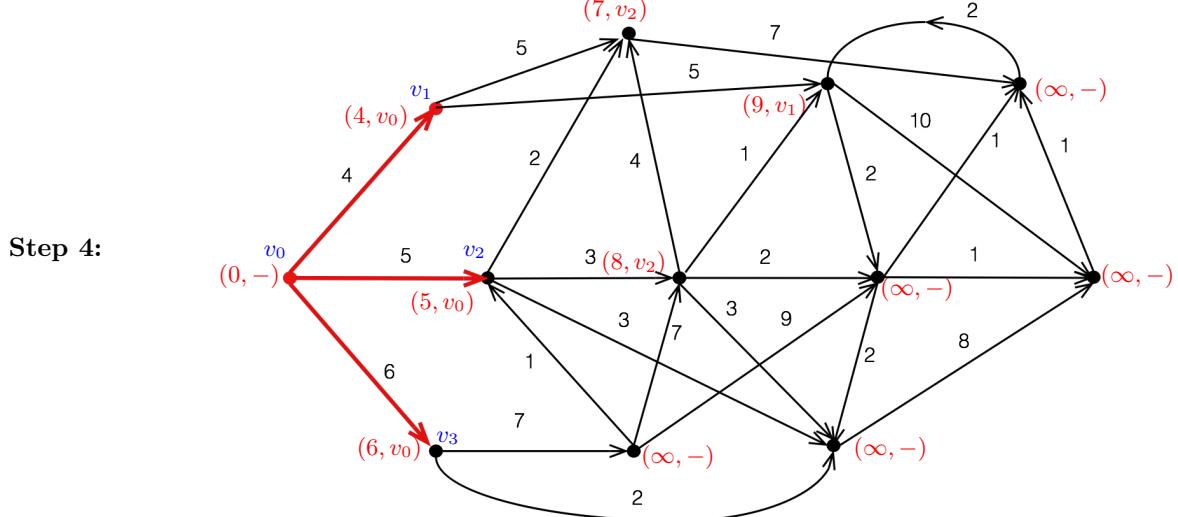
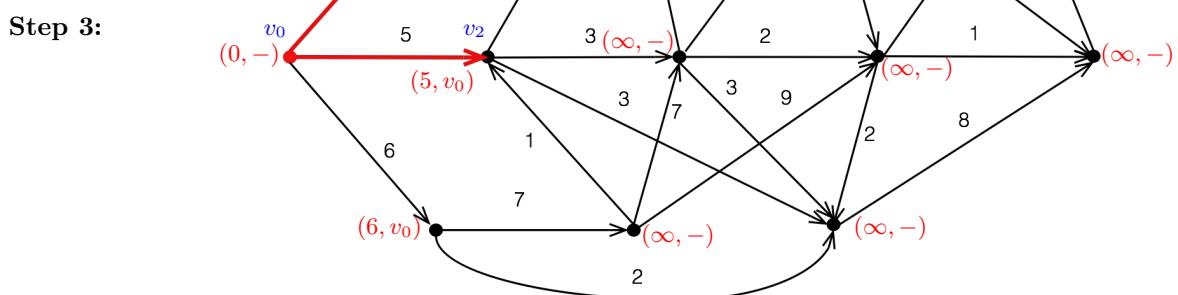
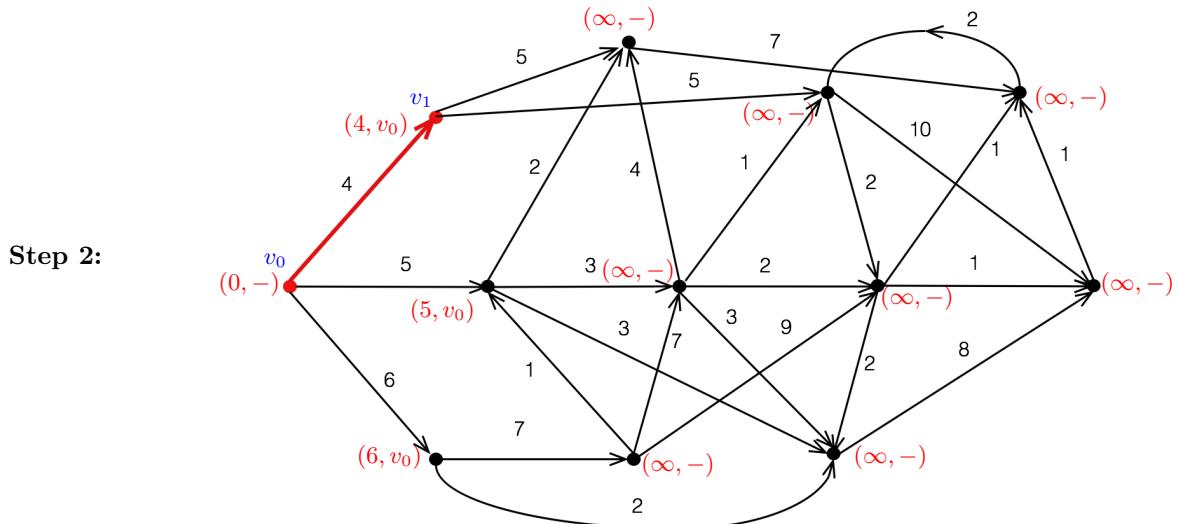
Set  $v_{j+1} := \bar{v}_*$ ,  $u_{j+1} := u$ ,  $\mathbf{L} := \mathbf{L} \cup \{(L(v_{j+1}), u_{j+1})\}$ .

Set  $S_{j+1} := S_j \cup \{v_{j+1}\}$ . Set  $j := j + 1$  and return to the Main loop.

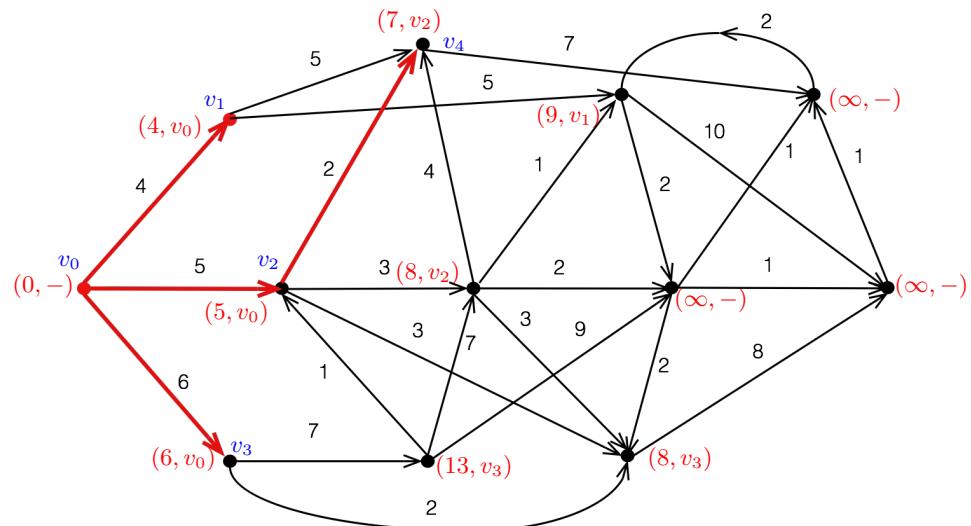
**Return L**

**Example.** Now describe the Dijkstra's Shortest Path Algorithm for the following digraph:

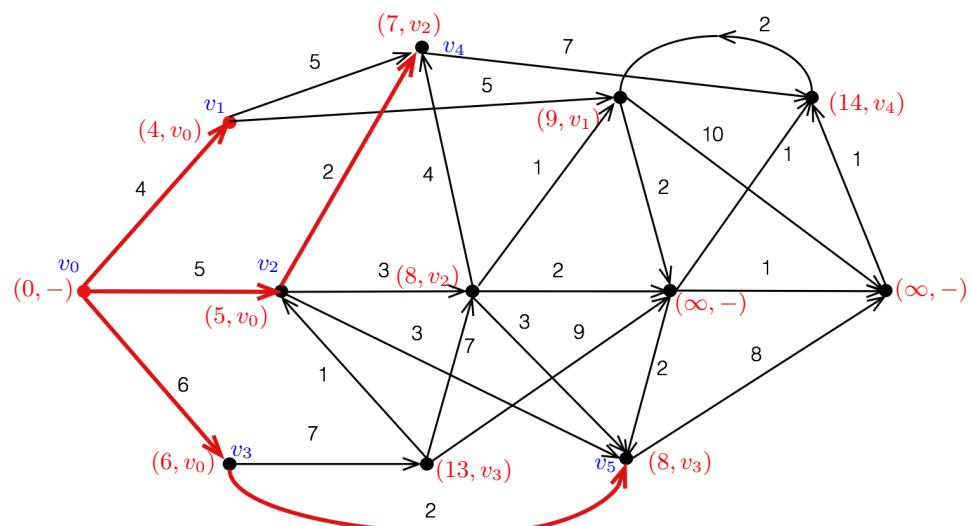




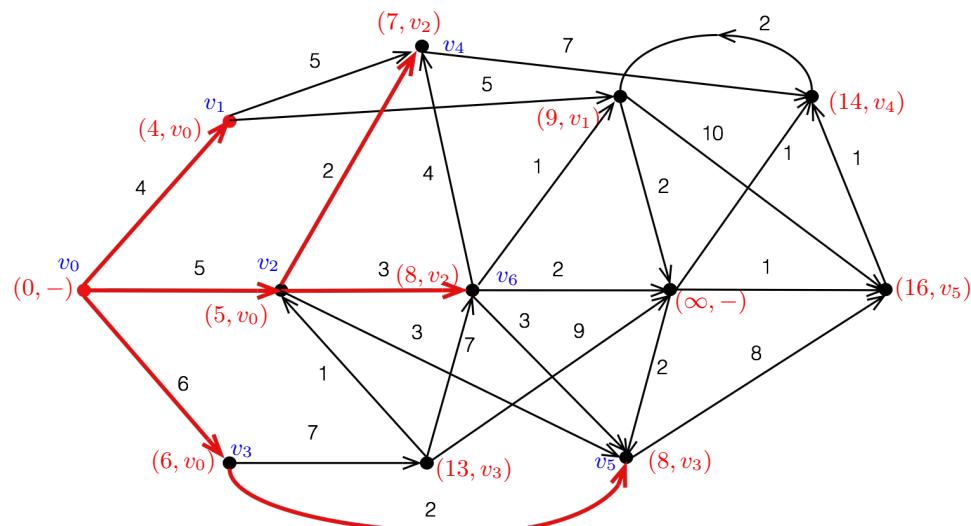
**Step 5:**



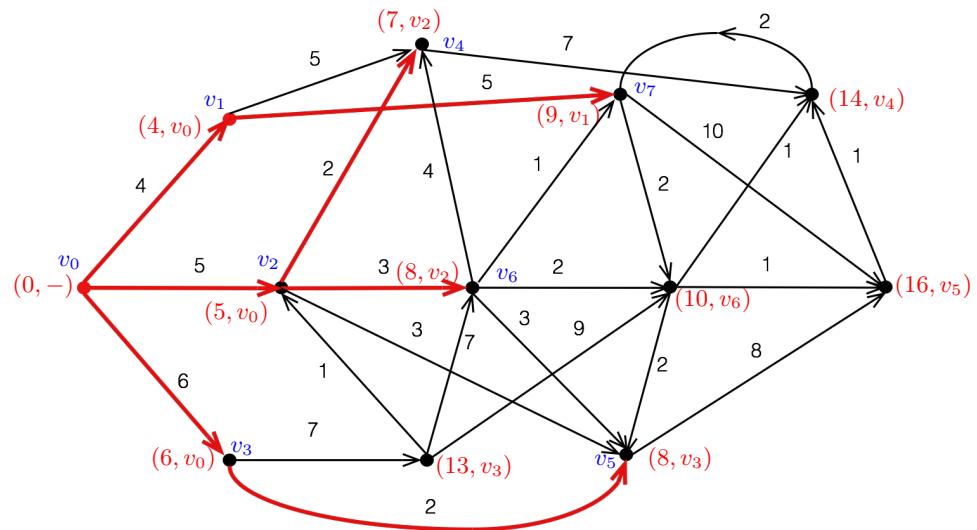
**Step 6:**



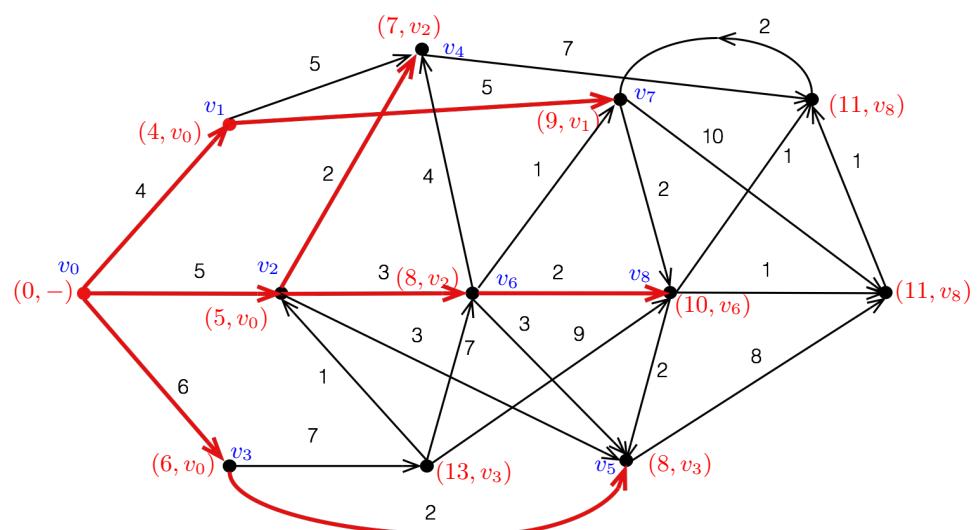
**Step 7:**



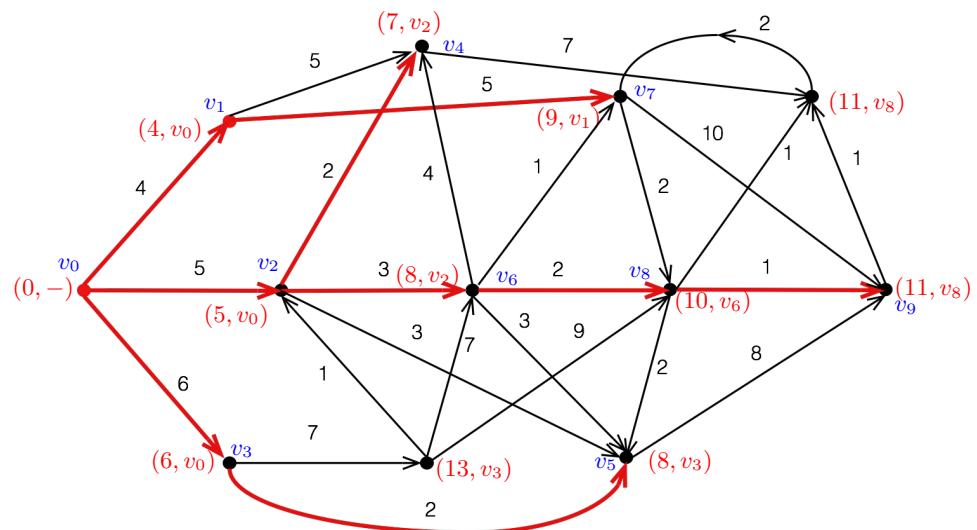
**Step 8:**



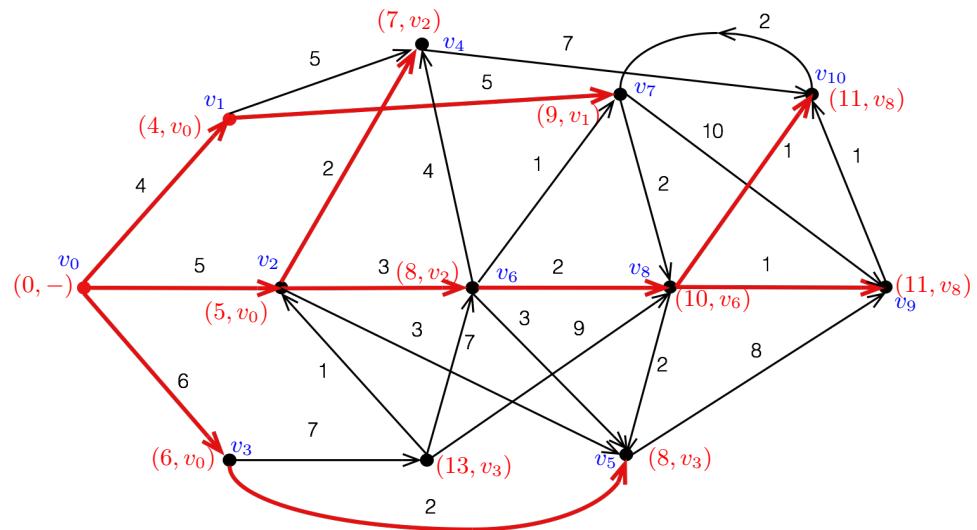
**Step 9:**



**Step 10:**



**Step 11:**



**Step 12:**

