FINAL EXAM REVIEW, PART II (MORE PROBLEMS ON MATHEMATICAL INDUCTION)

(1) Prove that $2n^3 + 3n^2 + n$ is divisible by 6 for any $n \geq 1$.

(2) Prove that $7^n - 2^n$ is divisible by 5 for any $n \geq 1$.

(3) Prove that $3^{2n+3} + 40n - 27$ is divisible by 64 for any $n \geq 1$.

(4) Prove that $5^{2n+1} \cdot 2^{n+2} + 3^{n+2} \cdot 2^{2n+1}$ is divisible by 19 for any $n \geq 1$.

(5) Prove the following identity for any $n \geq 1$:

$$1 + 3 + 5 + \cdots + (2n-1) = \sum_{i=1}^{n} (2i-1) = n^2.$$

(6) Prove the following identity for any $n \geq 1$:

$$1^2 + 4^2 + 7^2 + \cdots + (3n-2)^2 = \sum_{i=1}^{n} (3i-2)^2 = \frac{n(6n^2 - 3n - 1)}{2}.$$

(7) Prove the following identity for any $n \geq 1$:

$$2^2 + 5^2 + 8^2 + \cdots + (3n-1)^2 = \sum_{i=1}^{n} (3i-1) = \frac{n(6n^2 + 3n - 1)}{2}.$$

(8) Prove the following identity for any $n \geq 1$:

$$1 \cdot 2 + 2 \cdot 4 + \cdots + n(n+2) = \sum_{i=1}^{n} i(i+2) = \frac{n(n+1)(2n+7)}{6}.$$

(9) Prove that $3^{2n} - 1$ is divisible by 8 for any $n \geq 1$.

(10) Let $x \geq -1$. Prove that $(1+x)^n \geq 1 + nx$ for any $n \geq 1$.

(11) Prove that $n! < n^n$ for any $n \geq 2$.

(12) Prove the following identity for any $n \geq 1$:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{(n-1)n} = \sum_{i=1}^{n-1} \frac{1}{i(i+1)} = \frac{n-1}{n}.$$

(13) Prove the following identity for any $n \geq 1$:

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2n-1)(2n+1)} = \sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}.$$

(14) Prove the following identity for any $n \geq 1$:

$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right) \cdots \left(1 - \frac{1}{n}\right) = \prod_{i=2}^{n} \left(1 - \frac{1}{i}\right) = \frac{1}{n}.$$