This first Special Assignment is due on Tuesday, January 24. These Special Assignments should be viewed as mini-term papers; the goal is perfection. Not only should you aim to be absolutely correct, but you want to communicate flawlessly. There is no right format; use what fits you best. It can be heavy on prose and light on symbols or vice versa. But you want the presentation to be in English (applied to mathematics) with correct grammar. Of course using symbols like $\Rightarrow$ is fine, but do try to have them make grammatical sense.

There is one technical issue. Use only 8 1/2 by 11 inch note paper, write on only one side, and staple multiple pages together in the upper left corner. Finally, when you turn them in, do not fold the paper and be sure your name and assignment number are in the upper right hand corner.

Okay. Then here is your first Special Assignment:

1. Here is an important inequality, Bernoulli’s Inequality, that pops up often. It follows easily from the Binomial Theorem, but here we’d like a simpler direct proof. So, using induction, prove:

   Let $x > -1$. Prove that for all $n \in \mathbb{N}$

   $$(1 + x)^n \geq 1 + nx.$$ 

2. There are real numbers such as $\sqrt{2}$, that so bedeviled the Pythagoreans, that are “constructible” as the hypotenuse of an isosceles right triangle with sides of length 1, but are not rational. But unbeknownst to them there are even more puzzling real numbers, such as the familiar $\pi$ and $e$. Still these can be described in terms of rational numbers. Indeed, prove that

   $$\sup\{q \in \mathbb{Q} : q < \pi\} = \pi.$$ 

3. We are often faced with the task of deciding whether or not two real numbers $a$ and $b$ are equal. Here is a particularly useful technique for making such decisions.

   Let $t > 0$. Prove, using Archimedes, that if $a, b \in \mathbb{R}$ and

   $$|a - b| < t/n \quad \forall \ n \in \mathbb{N},$$

   then $a = b$. 