Assignment #8 — Math 315. Due: Friday, March 3.

[Note: The second 50 minute exam will be given on Friday, March 3. It will cover the material from Assignments 1 through 7.]

This week we will continue our study of general single valued functions of a single real variable — the sort of function that was the focus in elementary calculus. Specifically, we will aim to find out what limits of such functions really are! After our work on sequences and continuity, this should be fairly straightforward stuff. The one main wrinkle is that there are a great many variations on the notion, and so keeping them all straight is a mild challenge.

Your specific assignment for this week is from Section 20 from the Ross text from which you should tackle the following exercises:


And the special exercise (see below):

S1. S1. (abc), d,e, (fgh), i (j), k, (l,m,n).

S1. Let \( f \) be a function on \( \mathbb{R} \), and let \( a, L \in \mathbb{R} \). Below we have written the non-sequence (i.e., the “\( \varepsilon-\delta \)” definition of \( \lim_{x \to a} f(x) = L \)). Complete the list (see Discussion 20.9 in the Ross text) by writing out the non-sequence definition of the remaining fourteen principal cases below:

Sample \( \lim_{x \to a} f(x) = L \iff \forall \varepsilon > 0 \exists \delta > 0 \text{ such that } \forall x \in \mathbb{R} \setminus \{a\}, |x - a| < \delta \implies |f(x) - L| < \varepsilon. \)

(a) \( \lim_{x \to a^-} f(x) = +\infty \)  
(b) \( \lim_{x \to a^+} f(x) = L \)  
(c) \( \lim_{x \to a^+} f(x) = +\infty \)  
(d) \( \lim_{x \to a^-} f(x) = -\infty \)  
(e) \( \lim_{x \to a^-} f(x) = L \)  
(f) \( \lim_{x \to a^-} f(x) = +\infty \)  
(g) \( \lim_{x \to a^-} f(x) = -\infty \)  
(h) \( \lim_{x \to a^+} f(x) = -\infty \)  
(i) \( \lim_{x \to +\infty} f(x) = L \)  
(j) \( \lim_{x \to +\infty} f(x) = +\infty \)  
(k) \( \lim_{x \to +\infty} f(x) = -\infty \)  
(l) \( \lim_{x \to -\infty} f(x) = L \)  
(m) \( \lim_{x \to -\infty} f(x) = +\infty \)  
(n) \( \lim_{x \to -\infty} f(x) = -\infty \)