

## THE SPACE DENSITY OF GALAXIES THROUGH $\mu_B(0) = 25.0$ MAGNITUDES PER INVERSE ARCSECOND SQUARED

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Received 1999 September 2; accepted 1999 September 20

### ABSTRACT

Using the catalog of O'Neil, Bothun, & Schombert, we examine the central surface brightness distribution ( $\phi[\mu_B(0)]$ ) of galaxies in the  $22.0 \leq \mu_B(0) \leq 25.0$  mag arcsec $^{-2}$  range. Taking advantage of having a catalog in which each galaxy has a known central surface brightness, scale length, and redshift, we apply a bivariate volume correction to the data and extend the surface brightness distribution function by 1 magnitude, to 25.0 B mag arcsec $^{-2}$ . The result is a flat (slope = 0) space density of galaxies as a function of  $\mu_B(0)$  from the Freeman value of  $21.65 \pm 0.30$  to the survey limit of 25.0 mag arcsec $^{-2}$ , more than 10  $\sigma$  away. The galaxies that comprise this sample have scale lengths and circular velocities similar to  $L_*$  spiral galaxies (i.e., these are not dwarf galaxies). As such, a significant amount of mass is contained in this population. If the baryonic mass fraction of disk galaxies is independent of  $\mu_B(0)$ , then our results are consistent with a significant percentage of the baryonic content of the universe being contained in potentials only dimly lit by the embedded galaxy.

*Subject headings:* galaxies: fundamental parameters — galaxies: luminosity function, mass function — galaxies: spiral — galaxies: statistics

### 1. INTRODUCTION

Using the observed D/H ratio, nuclear cross sections, and the hot big bang cosmological model, the predicted baryon density of the universe is

$$0.013 < \Omega_B h_{100}^2 < 0.019$$

(i.e., Walker et al. 1991; Weinberg et al. 1997; Burles & Tytler 1996; Rauch et al. 1997). Estimating the known baryon mass density of the universe using  $\Omega_B = \Omega_{E/S0} + \Omega_{Sp} + \Omega_{clusters} + \Omega_{groups}$ , Persic & Salucci (1992) found  $\Omega_B = 2.2 + 0.6 h_{100}^{-3/2} \times 10^{-3}$ , showing that 70%–80% of the predicted baryon mass does not exist in standard galaxy catalogs. There are three possible locations for these “missing” baryons: (1) a cosmological background (possibly ionized), (2) the hot gas component of galaxy groups/clusters, and (3) galaxies that are difficult to detect and therefore missing from current catalogs.

At high redshift it appears that most of the baryons are either contained in damped Ly $\alpha$  systems that are neutral (Storrie-Lombardi, McMahon, & Irwin 1996) or the ionized Ly $\alpha$  forest. Modeling by Rauch et al. (1997) (see also Haehnelt, Steinmetz, & Rauch 1998) is consistent with the general notion that half of the baryons (e.g., hydrogen gas) are in the neutral phase and half are in the ionized phase that forms the Ly $\alpha$  forest. The critical issue now is the evolution of these structures and whether or not the abundance of this gas at high redshift can be reconciled with the abundance of galaxies at low redshift. The calculation of Persic & Salucci, which includes hot gas in galaxy clusters and groups, clearly shows that our baryon census falls well short of the big bang nucleosynthesis (BBNS) mark. However, a new census performed by Fukugita, Hogan & Peebles (1998) suggests we are far closer to the BBNS estimate than previously thought. They argue that most of the baryons are in the form of a hot, intergalactic gas (see also Cen &

Ostriker 1999). The relation of this component to the Ly $\alpha$  forest is unclear, but the implication of the Fukugita et al. study is that most baryons are not contained in galactic potentials. If so, this may leave little room for a large population of uncataloged galaxies.

In their review, Impey & Bothun (1997) summarize the last 10 years of effort to overcome the selection effects associated with the detectability of intrinsically diffuse galaxies (e.g., galaxies of low surface brightness—LSB). This effort has established that (1) LSB galaxies represent a different evolutionary path available to galaxies, (2) that LSB galaxies can be found at all galactic mass scales, (3) that some LSB galaxies likely formed relatively recently, and (4) that the space density of LSB galaxies is approximately  $10^6$  times higher than extrapolation of the Freeman (1970) law predicts. The major unresolved issues, difficult to determine observationally, are (1) how low in central surface brightness do disks galaxies go and (2) what is the overall space density of these objects. The latter point bears directly on the missing baryon issue and may tie into the evolution of the Ly $\alpha$  forest clouds if those structures have long cooling and collapse times. Indeed, Linder (1999) shows that it is highly plausible that most of the Ly $\alpha$  absorbers are associated with LSB galaxies.

Previous studies (e.g., McGaugh, Bothun, & Schombert 1995; Schwarzenberg et al. 1995; McGaugh 1996; Dalcanton et al. 1997) have shown that the space density of galaxies as a function of central surface brightness  $[\mu_B(0)]$  is either flat or slightly rising out to a limit of  $\mu_B(0) \sim 23.5$  mag arcsec $^{-2}$ . These results hold for disk galaxies (not dwarfs) of scale length larger than 1 kpc. Thus, there appear to be as many Freeman disks  $[\mu_B(0) \sim 21.5$  mag arcsec $^{-2}$ ] as there are disks with  $\mu_B(0)$  two magnitudes fainter. Given this, Impey & Bothun (1997) speculate that if the mass-to-light ratio of LSB galaxies is higher than in high surface brightness (HSB) galaxies, as is indicated in most

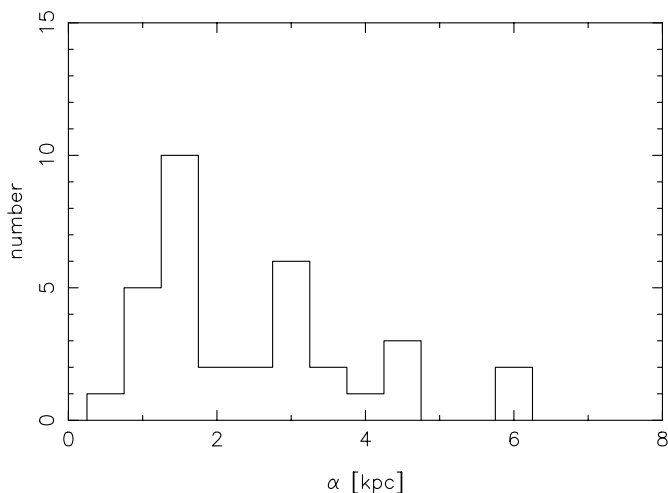


FIG. 1.—Distribution of scale lengths (in kpc) for the galaxy sample in this paper.

LSB galaxy studies (i.e., McGaugh & de Blok 1997) and if the faint end of the luminosity function has a slope of  $-1.6$  to  $-1.8$  (i.e., de Propris et al. 1995; Bothun, Impey, & Malin 1991), then the total contribution to the baryonic mass from galaxies is  $\Omega_B h_{100}^2 = 0.014\text{--}0.020$ , well within the bounds set by, e.g., Walker et al. (1991). This assumes that the space density of LSB galaxies remains flat through  $\mu_B(0) \sim 29$  mag arcsec $^{-2}$ . The point of this paper is to use new survey data to extend the previous limits by  $\sim 1.5$  mag in  $\mu_B(0)$  to probe further whether or not the space density of LSB galaxies remains flat.

## 2. A NEW LSB GALAXY SAMPLE

### 2.1. The Detectability of LSB Galaxies

Our ability to detect objects in the universe has been likened to standing in a well-lit room in the middle of the night and trying to look through the window to describe the garden 100 yards away. Although it may be possible to say definitively that the garden exists and even to describe some of the large, well-defined plants, coming up with a quantitative description of the fainter, or smaller, or more distant plants is an extremely difficult task. Moreover, of primary scientific interest is not the garden itself, but rather the evolutionary history of the plants that occupy it. With only this one view of the garden available to us, it would be extremely unlikely that our derived evolutionary history would be very accurate. The parallel between the garden and galaxy detection should be clear. In 1965 Arp attempted to quantify this limited view of the universe by defining a “band of visibility,” outside of which we are unable to discern galaxies, owing to either the small apparent size of the galaxies or their optically diffuse nature. Arp’s argument was later quantified by Disney (1976), showing that the visibility bias was rather severe.

Since Arp’s work was published, the “band of visibility” has been significantly broadened through improvements in both instruments and detection techniques. As an example, the superb angular resolution of *Hubble Space Telescope* (*HST*) has allowed for the distinction between true stars and galaxies that appear starlike in lower resolution surveys (e.g., Lilly et al. 1995; Ellis et al. 1996; Cowie et al. 1996; Steidel et al. 1996; Abraham et al. 1994; Morris et al. 1999;

O’Neil et al. 1998). The fact that most of these newly resolved galaxies are very far away means that the local universe is not filled up with little, dinky, high surface brightness (HSB) galaxies. On the other end of the spectrum, both improvements in detection techniques (i.e., Malin 1978; Schwartzberg et al. 1995) and the advent of CCD cameras as a tool in observing has allowed for the detection of increasingly diffuse (low signal-to-noise ratio) stellar systems (e.g., Impey, Bothun, & Malin 1988; Davies, Philipps, & Disney 1988; Dalcanton et al. 1997; Schombert & Bothun 1988; O’Neil, Bothun, & Cornell 1997a; Matthews & Gallagher 1997). Indeed the recent detection of extremely LSB dwarf spheroidal galaxies around Andromeda by Armandroff, Davies, & Jacoby (1998) is consistent with the local universe having a large population of low-mass, nearly invisible galaxies. The Andromeda discovery underscores the severity of surface brightness selection effects. Where once the Milky Way stood alone in the Local Group as a unique host of seven LSB dwarf spheroidals, we now have detected an apparently equivalent population around M31.

To establish the true space density of these newly discovered galaxies, we must make accurate corrections for the decreased probability of detecting a galaxy the closer it lies to the survey limits. The mathematical formalism of this correction has been extensively discussed in the literature (i.e., Disney 1976; Disney & Philipps 1983; McGaugh 1996; de Jong 1996). When applied to the available data (i.e., McGaugh et al. 1995; de Jong 1996; Dalcanton et al. 1997), these corrections yield the flat space density distribution discussed in § 1. We emphasize again that this distribution refers to *nondwarf* galaxies; i.e., those objects with scale lengths larger than  $\sim 1$  kpc.

### 2.2. The OBS Sample

In this paper we use the sample of LSB galaxies in the O’Neil, Bothun, & Schombert (1999a, hereafter OBS) catalog to determine the surface brightness distribution function from  $\mu_B(0)$  of 22.0–25.0 *B* mag arcsec $^{-2}$  (the catalog limits). This sample has well-defined survey limits and known  $\mu_B(0)$ , scale lengths, and redshifts, allowing for the use of a bivariate correction for the survey selection. The OBS sample is comprised of 43 galaxies with 21 cm measured redshifts with  $22.0 \leq \mu_B(0) < 25.0$  mag arcsec $^{-2}$  and with scale lengths ranging from 0.5 to 6.0 kpc (see Fig. 1). The galaxies are located in the direction of the Pegasus Cluster, Cancer Cluster, and near various known galaxies lying in the region of the Great Wall (i.e., Dell’Antonio, Geller, & Bothun 1996). As detailed in OBS, the galaxies span a large range in color, size, and circular velocity, with the majority having characteristics similar to those of  $L_*$  spirals. Thus this sample will allow for an accurate determination of the space density of galaxies with  $\mu_B(0)$  in the range of 23.0–25.0 *B* mag arcsec $^{-2}$ , substantially fainter than previous studies.

## 3. THE VOLUME CORRECTION

As has often been discussed, detecting HSB galaxies within a survey is considerably easier than detecting LSB galaxies (i.e., Freeman 1970; Disney 1976; Disney & Philipps 1983; McGaugh et al. 1995; Davies 1990; de Jong 1996; Bothun, Impey, & McGaugh 1997; Dalcanton et al. 1997; O’Neil et al. 1997b). Thus determining the true (underlying) surface brightness distribution of a sample of galaxies requires accounting for the probability of a galaxy

being detected by a survey of a given design. For a field galaxy survey, the probability of detection is determined simply by the available volume that can be sampled for a galaxy of a given size and luminosity (e.g., its surface brightness). The volume-corrected surface brightness distribution is thus

$$\phi(\mu_0) = \sum_{i=1}^N \frac{S^i}{V_{\max}^i}, \quad (1)$$

where  $i$  is summed over all  $N$  galaxies in the sample,  $S^i$  is 0 or 1 depending on whether a galaxy lies within the described volume, and  $V_{\max} = (4\pi/3)d_{\max}^3$ , the maximum volume in which a galaxy could be detected.

For a surface brightness-limited sample (i.e., OBS),  $d_{\max}$  can be found by requiring that the diameter of the galaxy be equal to, or greater than, the minimum detectable diameter ( $\theta = 2r \geq \theta_i$ ). For a galaxy with an exponential surface brightness profile this gives

$$\mu(r) = \mu_0 + 1.086 \frac{r}{\alpha}, \quad (2)$$

$$\theta = 2r = 1.84\alpha(\mu_i - \mu_0) \propto \frac{h}{d}(\mu_i - \mu_0), \quad (3)$$

$$d_{\max}(\mu_0) \propto \frac{h}{\theta_i}(\mu_i - \mu_0), \quad (4)$$

where  $\mu(0)$  is the central surface brightness of the galaxy,  $\alpha$  is its scale length in arcseconds, and  $h$  is its scale length in kiloparsecs. Thus, for a surface brightness-limited sample,

$$V_{\max}(\mu_0) \propto \left(\frac{h}{\theta_i}\right)^3 (\mu_i - \mu_0)^3 \propto \left(\frac{\alpha d}{\theta_i}\right)^3 (\mu_i - \mu_0)^3. \quad (5)$$

#### 4. THE SURFACE BRIGHTNESS DISTRIBUTION, $\phi(\mu_0)$

Figure 2 shows the results of applying the correction given in equation (5) to the O'Neil et al. (1997a) data using the redshifts available in OBS. The limiting diameter was set to  $25''$ , and  $\mu_l = 25.0$  mag arcsec $^{-2}$ . This corresponds to an approximate minimum physical diameter of 3 kpc (so again, these are *nondwarf* galaxies). For the OBS survey, the limiting central surface brightness was found through an extensive series of computer models, in which Monte Carlo-type simulations of the images were created and searched for galaxies (O'Neil et al. 1997b). As the true underlying galaxy distribution of the computer-generated images was known, the detection cutoff could be well determined. Thus,  $\mu_l$  and  $V_{\max}$  for the OBS catalog are also well determined.

Because the OBS sample is not uniformly distributed in space but instead follows the same large scale-structure as the HSB galaxies in the region (i.e., Fig. 2 of OBS), performing a  $V/V_{\max}$  test on the galaxies, and normalizing the distribution function to that (i.e., de Jong 1996), would be extremely difficult and possibly misleading at best. In practice, the OBS sample lies in a shell bounded by radial velocities of 4000 and 12,000 km s $^{-1}$ . The data for this sample, as well as for the comparison samples, have therefore been normalized to one (Fig. 2). Additionally, to ensure against bias due to undersampling within a bin, the data from OBS were binned to 0.5 mag arcsec $^{-2}$ . The errors bars for this data are simply  $\sqrt{N}/N$ . The low values for the surface brightness distribution between 22 and 23 mag arcsec $^{-2}$  are artificial, caused by the 22.0  $B$  mag arcsec $^{-2}$  cutoff in the survey sample imposed in the OBS catalog. This was not corrected for. The total number of galaxies in each bin, then, is  $22.25 \leq \mu_B(0) < 22.75$ , six galaxies;  $22.75 \leq \mu_B(0) < 23.25$ , seven galaxies;  $23.25 \leq \mu_B(0) < 23.75$ , 18 galaxies;  $23.75 \leq \mu_B(0) < 24.25$ , five galaxies;  $24.35 \leq \mu_B(0) < 24.75$ ,

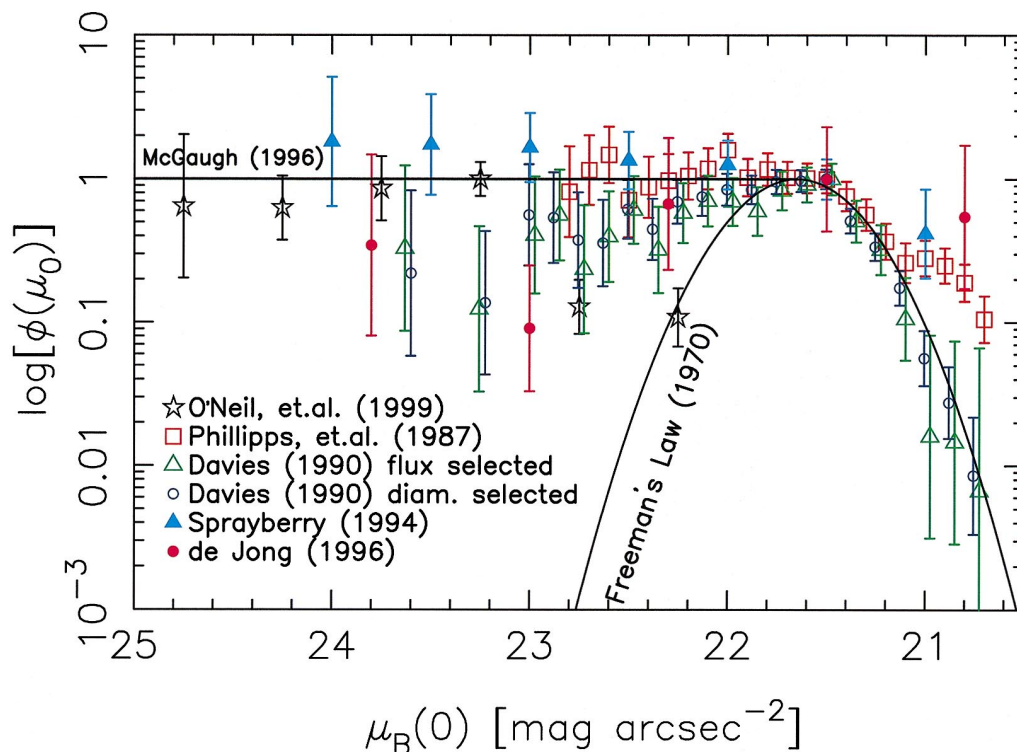


FIG. 2.—Volume-corrected surface brightness distribution function for all the galaxies in this and other surveys

five galaxies; and  $24.75 \leq \mu_B(0)$ , one galaxy. Although there is only one galaxy in the last bin [ $24.75 \leq \mu_B(0)$ ], the fact that any galaxies were found so close to the survey cutoff is highly significant, as the probability of its detection is extremely low.

The data from this survey extend the faint end of the surface brightness distribution function in a horizontal line from  $23.0 \text{ mag arcsec}^{-2}$  through  $25.0 \text{ mag arcsec}^{-2}$ , the survey cutoff, matching the predictions made by, i.e., McGaugh (1996) and Impey & Bothun (1997). A flat distribution here, however, does not imply that most of the luminosity density in the universe is contained in LSB galaxies for the simple reason that at a given scale length, LSB galaxies are less luminous compared to HSB galaxies. Arguments given in McGaugh (1998) show that even for a flat space density of LSB disks,  $\leq 30\%$  of the total extragalactic light is contained in that population.

Our flat space distribution does contradict some of the data points of both de Jong (1996) and Davies (1990) in the  $23.0\text{--}24.0 \text{ mag arcsec}^{-2}$  range, where their counts appear to dip downward. The first, and most obvious, explanation for the discrepancy is that the volumetric corrections for one or more of these samples was done incorrectly, either owing to misidentifying the selection limits of the survey or through poor statistical sampling. The OBS sample is designed to look for galaxies above the  $22.0 \text{ mag arcsec}^{-2}$  range and is therefore complete through  $24.0 \text{ mag arcsec}^{-2}$ , with the uncorrected data having a flat surface brightness distribution from  $22.5$  through  $25.0 \text{ mag arcsec}^{-2}$  (i.e., Fig. 8 of O'Neil 1998). Additionally, the surface brightness and diameter cutoff for the OBS sample was determined through computer modeling (O'Neil et al. 1997b) and therefore is well determined. In contrast, the de Jong sample ranges in central surface brightness from approximately  $20.0$  through

$24.1 \text{ mag arcsec}^{-2}$ , with the majority of the galaxies lying in the  $21.0\text{--}22.0 \text{ mag arcsec}^{-2}$  range. The volumetric corrections for the de Jong sample are concerned only with the diameter limit ( $\theta_l$ ) imposed on the survey and do not account for potential surface brightness selection effects, an omission that de Jong states could cause his survey to be undersampled at the faint  $\mu(0)$ , large scale length and/or small scale length, bright  $\mu(0)$  ends of the spectrum. Combined with the survey's undersampling in the  $\mu(0) > 22.5 \text{ mag arcsec}^{-2}$  range, this could result in an artificial drop in the de Jong surface brightness distribution.

Like the de Jong (1996) sample, the Davies (1990) sample is concerned with the entire range of central surface brightnesses, but in this case the total number of galaxies involved in the survey should preclude any difficulties with undersampling. The Davies sample was corrected for surface brightness selection effects, with a limiting central surface brightness  $\mu_{\text{lim}}(0) = 25.6 \text{ mag arcsec}^{-2}$ , and  $\theta_l = 7''$ . This low value for  $\theta_l$  potentially mixed dwarfs and nondwarfs together, which could greatly confuse the situation. More importantly, no galaxies were actually detected near the defined survey limits. Thus it is entirely possible that the chosen sample limits simply do not accurately reflect the nature of the survey and thus are inappropriate in determining the volumetric correction. This could account for the apparent undersampling in the  $23.25 \text{ mag arcsec}^{-2}$  bin compared to our data. Figure 3 shows the results of changing the binning for the OBS sample, from bins of  $1.0 \text{ mag arcsec}^{-2}$  through bins of  $0.3 \text{ mag arcsec}^{-2}$ . The behavior of the surface brightness distribution as the data become undersampled imitates the behavior of the de Jong and Davies samples. It is therefore possible that, as both the de Jong and Davies samples are primarily HSB galaxy samples, they are relatively undersampled in LSB galaxies.

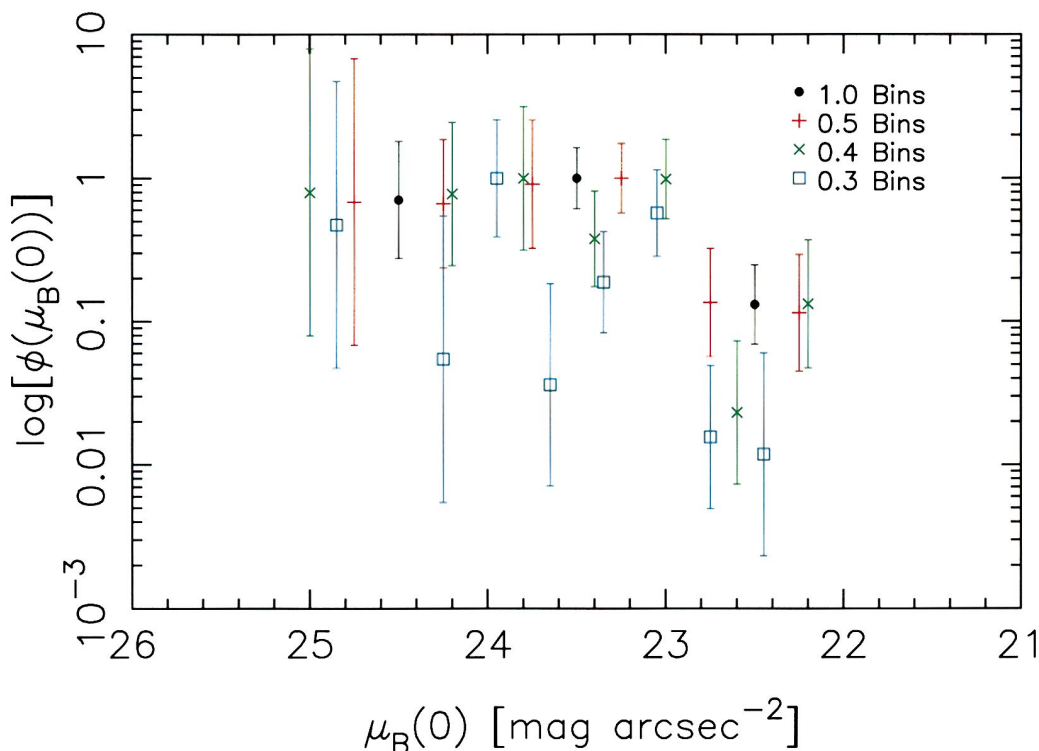


FIG. 3.—Effects of undersampling the data in this survey. The circles, plus signs, crosses, and squares result from binning the galaxies in 1.0, 0.5, 0.4, and 0.3  $\text{mag arcsec}^{-2}$  bins, respectively.

A final possibility is that the difference in the samples is a reflection of some true variance in surface brightness distributions, caused by environmental factors, within the local universe. That is, as only a (relatively) small spatial volume was examined for each survey, we may be viewing local differences in the surface brightness distribution. If this is the case, the true surface brightness distribution of the local universe may lie somewhere between our sample and the de Jong and Davies samples. An all-sky survey is needed to rectify this, but it remains unclear, at this point, if the automatic image detection software of the Sloan Digital Sky Survey will in fact detect LSB galaxies on the first pass (M. Strauss 1998, private communication). While not an all-sky survey, O'Neil et al. (1997a) did take care to sample a range of environments in order to minimize any possible variances caused by environmental factors.

At this point the importance of the chosen value for  $\mu_{\text{lim}}$  should also be noted. Choosing a value that is fainter (or brighter) than the true survey limits will result in an artificial lowering (raising) of the surface brightness distribution slope at faint  $\mu_0$ . This is not surprising, as it simply is a statement that if a survey is believed to extend to, say, 26.0 mag arcsec<sup>-2</sup> and yet detects no objects with  $\mu_0 \geq 25.0$  mag arcsec<sup>-2</sup>, it would be accurate to assume that a falloff in galaxy number density at faint [ $\mu_0(0) \geq 25.0$ ] surface brightness is occurring. The OBS sample is no exception to this rule. Were  $\mu_{\text{lim}}$  reduced to 26.0, a slight decline in the slope of the surface brightness distribution function, beginning at 24.0 mag arcsec<sup>-2</sup>, would be evident. As  $\mu_{\text{lim}}$  was carefully determined for the OBS sample, though, it should be an accurate representation of the survey's true limitations. With this and the above considerations in mind, it is likely the flat surface brightness distribution given by the OBS sample through 23.5–25.0 mag arcsec<sup>-2</sup> is an accurate representation of the surface brightness distribution in the local ( $z < 0.03$ ) universe. The implication of such a flat distribution remains profound in terms of whether or not most of the baryons are located within a galaxy potential or are distributed in some cosmological background.

## 5. CONCLUSION

Using a bivariate volume correction, we extend the surface brightness distribution function in a horizontal line from the Freeman value of 21.65  $B$  mag arcsec<sup>-2</sup> through 25.0  $B$  mag arcsec<sup>-2</sup>, the limit of the OBS catalog. This result is consistent with previous studies (e.g., McGaugh et al. 1995; Dalcanton et al. 1997) but extends them to fainter surface brightness levels. Our result is somewhat inconsistent with the findings of two previous surveys in the 23.0–24.0  $B$  mag arcsec<sup>-2</sup> range (Davies 1990; de Jong 1996). However, our survey was designed specifically to detect galaxies in this surface brightness range, and we have quite well defined survey limits ( $\theta_l, \mu_l$ ). This leads us to have considerable confidence in our principal result that the space density of disk galaxies as a function of  $\mu_B(0)$  remains flat out to values of  $\mu_B(0) = 25.0$  mag arcsec<sup>-2</sup>.

The surface brightness distribution as predicted by Freeman's law is  $\mu_B(0) = 21.65 \pm 0.35$  mag arcsec<sup>-2</sup>. We are 10  $\sigma$  fainter than this value, and our data indicate an equal space density of galaxies at this level. For every galaxy with  $\mu_B(0) = 21.65$ , there is one with  $\mu_B(0) = 22.0, 23.5,$  and  $25.0$  mag arcsec<sup>-2</sup>, and each of these galaxies has similar circular velocities. Hence, the LSB galaxies are not preferentially less massive than their higher surface brightness counterparts (e.g., OBS; McGaugh & de Blok 1997; Schombert et al. 1992; O'Neil, Verheijen, & McGaugh 1999b). As a result, our data are highly consistent with the proposition put forth in Impey & Bothun (1997) that much of the apparent missing baryon problem is resolved with a flat distribution of the space density of galaxies as a function of  $\mu_B(0)$ . That is, a lot of baryons are contained in potentials that host galaxies that are very diffuse and hard to detect. Whether the space density of LSB galaxies remains flat through  $\mu_B(0) = 28.0$  mag arcsec<sup>-2</sup> or falls off after  $\mu_B(0) = 26.0$  mag arcsec<sup>-2</sup> awaits deeper data to determine. Given the extant data, we would not venture a prediction on this important matter.

## REFERENCES

- Abraham, R. G., Valdes, F., Yee, H., & van den Bergh, A. 1994, *ApJ*, 432, 75  
 Armandroff, T., Davies, J., & Jacoby, G. 1998, *AJ*, 110, 2287  
 Arp, H. 1965, *ApJ*, 142, 402  
 Bothun, G., Impey, C., & Malin, D. 1991, *ApJ*, 376, 404  
 Bothun, G., Impey, C., & McGaugh, S. 1997, *PASP*, 109, 745  
 Burles, S., & Tytler, D. 1996, *Nature*, 381, 207  
 Cen, P., & Ostriker, J. 1999, *ApJ*, 514, 1  
 Cowie, L. L., Songalia, A., Hu, E. M., & Cohen, J. G. 1996, *AJ*, 112, 839  
 Dalcanton, J., et al. 1997, *AJ*, 114, 635  
 Davies, J. 1990, *MNRAS*, 244, 8  
 Davies, J., Phillipps, S., & Disney, M. 1988, *MNRAS*, 231, 69  
 de Jong, R. 1996, *A&A*, 313, 46  
 Dell'Antonio, I., Geller, M., & Bothun, G. 1996, *AJ*, 112, 1780  
 de Propriis, R., Pritchet, C., Harris, W., & McClure, R. 1995, *ApJ*, 450, 524  
 Disney, M. 1976, *Nature*, 263, 573  
 Disney, M., & Phillipps, S. 1983, *MNRAS*, 205, 1253  
 Ellis, R. S., Colless, M., Broadhurst, T., Heyl, J., & Glazebrook, K. 1996, *MNRAS*, 280, 235  
 Freeman, K. 1970, *ApJ*, 160, 811  
 Fukugita, M., Hogan, C. J., & Peebles, P. J. E. 1998, *ApJ*, 505, 518  
 Haehnelt, M., Steinmetz, M., & Rauch, M. 1998, *ApJ*, 495, 647  
 Impey, C., & Bothun, G. 1997, *ARA&A*, 35, 267  
 Impey, C., Bothun, G., & Malin, D. 1988, *ApJ*, 330, 634  
 Lilly, S. J., Le Fèvre, O., Crampton, D., Hammer, F., & Tresse, L. 1995, *ApJ*, 455, 50  
 Linder, S. 1999, preprint  
 Malin, D. 1978, *Nature*, 276, 591  
 Matthews, L., & Gallagher, J. 1997, *AJ*, 114, 1899  
 McGaugh, S. 1996, *MNRAS*, 280, 337  
 ———. 1998, in *ASP Conf. Ser. 182, Galaxy Dynamics*, ed. D. R. Marritt, M. Valluri, & J. A. Sellwood (San Francisco: ASP), 528  
 McGaugh, S., Bothun, G., & Schombert, J. 1995, *AJ*, 110, 573  
 McGaugh, S., & de Blok, W. J. G. 1997, *MNRAS*, 290, 533  
 Morris, S., et al. 1999, *ApJ*, in press  
 O'Neil, K. 1998, Ph.D. thesis, Univ. of Oregon, Eugene  
 O'Neil, K., Bothun, G., & Cornell, M. 1997a, *AJ*, 113, 1212  
 O'Neil, K., Bothun, G., & Schombert, J. 1999a, preprint (OBS)  
 O'Neil, K., et al. 1997b, *AJ*, 114, 2448  
 ———. 1998, *AJ*, 116, 657  
 O'Neil, K., Verheijen, M., & McGaugh, S. 1999b, *AJ*, submitted  
 Persic, M., & Salucci, P. 1992, *MNRAS*, 258, 14  
 Rauch, M., et al. 1997, *ApJ*, 489, 7  
 Schombert, J., et al. 1992, *AJ*, 103, 1107  
 Schombert, J., & Bothun, G. 1988, *AJ*, 95, 1389  
 Schwartzberg, J., et al. 1995, *MNRAS*, 275, 121  
 Steidel, C., Giavalisco, M., Pettini, M., Dickinson, M., & Adelberger, K. 1996, *ApJ*, 462, 17  
 Storrie-Lombardi, L., McMahon, R., & Irwin, M. 1996, *MNRAS*, 283, L79  
 Walker, T., et al. 1991, *ApJ*, 376, 51  
 Weinberg, S., et al. 1997, *ApJ*, 490, 564