Math 243: An example of a matched pairs t procedure

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Comments on exam problems

Always pay attention to whether the data were properly collected!

Descriptions of experiments requiring a placebo must make clear what a placebo is.

A correct solution to a hypothesis test must state the conclusion in terms of the original language of the problem. Just saying that we reject the null hypothesis, or conclude that $\mu < \mu_0$ (or something similar), or, if appropriate, that we do not reject the null hypothesis, etc., will not get full credit.

An example of a matched pairs t procedure

Professor Slemmox believes that female spiral-horned snorkacks have shorter horns than male spiral-horned snorkacks.

He intends to find a simple random sample of brother-sister pairs of spiral-horned snorkacks, and compare their horn lengths.

Hypotheses

This is a matched pairs design.

Let μ be the mean amount by which the horn lengths of female spiral-horned snorkacks exceed the horn lengths of male spiral-horned snorkacks.

H_0: $\mu = 0$.
H_a: $\mu < 0$.

Hypotheses: another version

We could also have done things the other way around. We could take μ to be the mean amount by which the horn lengths of male spiral-horned snorkacks exceed the horn lengths of female spiral-horned snorkacks. Then the hypotheses would be:

$\mathrm{H_0}$: $\mu = 0$.
$\mathrm{H_a}$: $\mu > 0$.

The sample

Use the first one:
H_0: $\mu = 0$.
H_a: $\mu < 0$.

We decide *now* on a significance level to require. Professor Slemmox (more importantly, his audience) doesn't know any obvious reason that this hypothesis implausible, so he chooses, say, $\alpha = 0.05$.

With considerable magical assistance, Professor Slemmox manages to locate 14 brother-sister pairs of spiral-horned snorkacks. We treat these as a simple random sample.

The data

Here are the horn lengths, measured in inches:

Pair	Female horn length	Male horn length
1	52	47
2	46	51
3	48	48
4	71	65
5	62	69
6	47	55
7	55	56
8	54	65
9	47	51
10	40	43
11	44	50
12	50	49
13	51	59
14	60	62

Treatment of the data

We apply the one sample t procedure to the differences in the pairs.

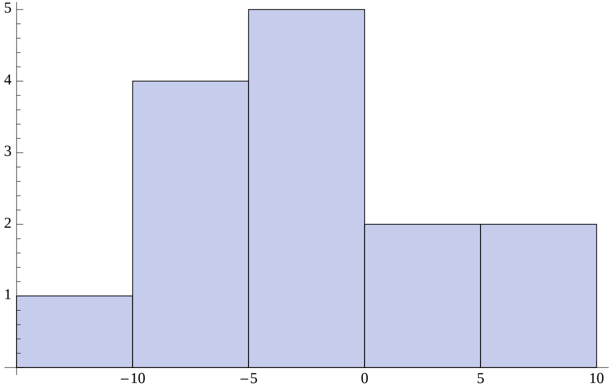
Differences

Here is the previous table, with the differences added:

Pair	Female horn length	Male horn length	Difference
1	52	47	5
2	46	51	−5
3	48	48	0
4	71	65	6
5	62	69	−7
6	47	55	−8
7	55	56	−1
8	54	65	−11
9	47	51	−4
10	40	43	−3
11	44	50	−6
12	50	49	1
13	51	59	−8
14	60	62	−2

Is the test safe to use?

We believe the data can be treated as a simple random sample. The sample size is small, so we have to check that the data are single peaked, roughly symmetric, and that there are no outliers. A stemplot was a bit awkward, so here is a histogram:



Is the test safe to use? (Continued.)

It seems reasonable.

A stemplot, in case you want to see:

−10	
−10	1
−0	6 7 8 8
−0	1 2 3 4 5
0	0 1
0	5 6

The statistic

My computer gave:

$$\overline{x} \approx -3.07143.$$

$$s \approx 4.95308.$$

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{\overline{x} - 0}{s/\sqrt{14}} \approx -2.32022.$$

$$P \approx 0.018617.$$

If you use Table C, use the row for 13 degrees of freedom $(n-1)$. Look for 2.32022. It is between 2.282 (for one-sided P of 0.02) and 2.650 (for one-sided P of 0.001). These numbers are the right tail probability.
We want the left tail probability at -2.32022. By symmetry, this is the same as the right tail probability at 2.32022, so is between 0.02 and 0.01.

Reject the null hypothesis

So we reject the null hypothesis.

In language related to the original problem

We conclude, at the significance level $\alpha = 0.05$, that female spiral-horned snorkacks do indeed have shorter horns than male spiral-horned snorkacks.