MATH 243 (PHILLIPS, SPRING 2008): SAMPLE FINAL EXAM PROBLEMS

General comments:

- The final exam is cumulative. The material from after the second midterm will be emphasized, but is still likely to be less than half the exam.
- Besides these problems, see all the material related to the midterms, the final exam from the previous course, the homework, the quizzes, the examples in the book, etc.
- Many variations of these problems are possible. In particular, the questions could be asked in different ways, something which is multiple choice here could be short answer on the real exam, and the scatterplots and histograms could have very different features. For example, a problem might ask something about a stemplot, or for information about quartiles instead of the mean, etc. Thus, a problem could test the same ideas but superficially not look like anything here.
- There will be something on the exam which is not covered in the sample problems.
- The number of problems of a given type which appear here need not represent the weight given to a topic on the actual midterm. The choice of problem here is partly intended to reflect the possible variety of questions on the exam. (Besides problems directly related to the material of the last two weeks of the course, I have also included some problems from the make-up versions of Midterms 1 and 2. These are for those who want extra practice on earlier material. However, they may **not** be representative of the problems from earlier material which actually appear on the final exam.)
- Recall that you may bring one 8.5×11 page of notes and a calculator; see the syllabus for details.
- Recall that **no** calculator help will be provided on the exam. If you need to, write instructions for your calculator on your 8.5×11 page of notes.

Some specific comments on the problems (not necessarily complete):

- **Always** pay attention to whether the data were properly collected!
- Descriptions of experiments requiring a placebo must make clear what a placebo is.
- Normal probability problems may ask for reasonably well drawn graphs.
- A correct solution to a hypothesis test must state the conclusion in terms of the original language of the problem. Just saying that we reject the null hypothesis, or conclude that $\mu < \mu_0$ (or something similar), or, if appropriate, that we do not reject the null hypothesis, etc., will not get full credit.
- Solutions to confidence interval problems must show the critical value you used (z^* or t^*).

1. Multiple choice problems

The following information is used in the next five questions.

An inspector inspects large truckloads of potatoes to determine the proportion p in the shipment with major defects prior to using the potatoes to make potato chips. She selects a simple random sample of 50 potatoes from the over 2000 potatoes on a particular truck. Only 2 of the potatoes sampled are found to have major defects.

MC2. What is the population of interest in the above data?

a. All potatoes grown in the continental U.S.
b. All potatoes that will be used to make potato chips.
c. All potatoes grown on farms owned by the company that makes the chips.

Date: 8 June 2008.

d. All potatoes on the truck.
e. The 50 potatoes in the sample.
f. None of the above.

MC3. Using the above data, what is the sample proportion $\widehat{p}$ of potatoes having major defects?

a. 0.04
b. 0.2%
c. 2.5%
d. 2
e. None of the above.

MC4. Using the above data, which of the following assumptions for inference about a proportion using a confidence interval based on $\widehat{p}$ are violated?

a. The data are a simple random sample from the population of interest.
b. The population much bigger than the sample.
c. n is so large that both the count of successes $n\widehat{p}$ and the count of failures $n\left(1-\widehat{p}\right)$ are 15 or more.
d. There appear to be no violations.
e. None of the above.

MC5. Using the above data, which of the following assumptions for inference about a proportion using the "plus four" 95% confidence interval are violated?

a. The data are a simple random sample from the population of interest.
b. The population much bigger than the sample.
c. The sample size is at least 10.
d. The required confidence is at least 90%.
e. There appear to be no violations.
f. None of the above.

MC6. Using the above data, suppose instead you wished to test the hypotheses:

$$\mathrm{H}_0 : p = 0.20.$$
$$\mathrm{H}_\mathrm{a} : p < 0.20.$$

Which of the following assumptions for inference about a proportion using a hypothesis test is violated?

a. The data are a simple random sample from the population of interest.
b. The population much bigger than the sample.
c. n is so large that both np_0 and $n(1-p_0)$ are 10 or more.
d. There appear to be no violations.
e. None of the above.

MC7. A simple random sample of 50 undergraduates at Johns Hopkins University found that 60% of those sampled felt that drinking was a problem among college students. A simple random sample of 50 undergraduates at Ohio State University found that 70% felt that drinking was a problem among college students. The number of undergraduates at Johns Hopkins University is approximately 2000, while the number at Ohio State is approximately 40,000. Which of the following is the most accurate statement?

a. The sample from Johns Hopkins has <u>much less</u> sampling variability than that from Ohio State.
b. The sample from Johns Hopkins has <u>much more</u> sampling variability than that from Ohio State.

c. The sample from Johns Hopkins has about the same sampling variability as that from Ohio State.
e. The sample from Johns Hopkins must have exactly the same sampling variability as that from Ohio State.
d. The sample from Johns Hopkins could have about the same sampling variability as that from Ohio State, or much more sampling variability, but it is impossible for it to have much less sampling variability.
f. It is impossible to make any statements about the sampling variability of the two samples since the students surveyed were different.

MC8. A simple random sample of 50 undergraduates at Johns Hopkins University found that 60% of those sampled felt that drinking was a problem among college students. Suppose the actual proportion of undergraduates at Johns Hopkins University who feel drinking is a problem among college students is 70%. The mean of the sampling distribution of the percentage that feel drinking is a problem in repeated simple random samples of 50 Johns Hopkins undergraduates is what?

a. 50%
b. 60%
c. 65%
d. 70%
e. Cannot be determined from the information given.
f. None of the above.

MC9. The number of undergraduates at Johns Hopkins University is approximately 2000, while the number at Ohio State University is approximately 40,000. At both schools a simple random sample of about 3% of the undergraduates is taken. At Johns Hopkins University, 60% of those sampled felt that drinking was a problem among college students. At Ohio State University, 70% felt that drinking was a problem among college students. Which of the following is the most accurate statement?

a. The sample from Johns Hopkins has much less sampling variability than that from Ohio State.
b. The sample from Johns Hopkins has much more sampling variability than that from Ohio State.
c. The sample from Johns Hopkins has about the same sampling variability as that from Ohio State.
e. The sample from Johns Hopkins must have exactly the same sampling variability as that from Ohio State.
d. The sample from Johns Hopkins could have about the same sampling variability as that from Ohio State, or much more sampling variability, but it is impossible for it to have much less sampling variability.
f. It is impossible to make any statements about the sampling variability of the two samples since the students surveyed were different.

MC10. In an effort to determine whether female intercollegiate soccer players are on average taller than the general population of undergraduate female college students, I choose a simple random sample of female intercollegiate soccer players and an independent simple random sample of undergraduate female college students. I compute the mean heights in both samples, and use the two sample t procedure to test the hypothesis that the soccer players are taller. This study is:

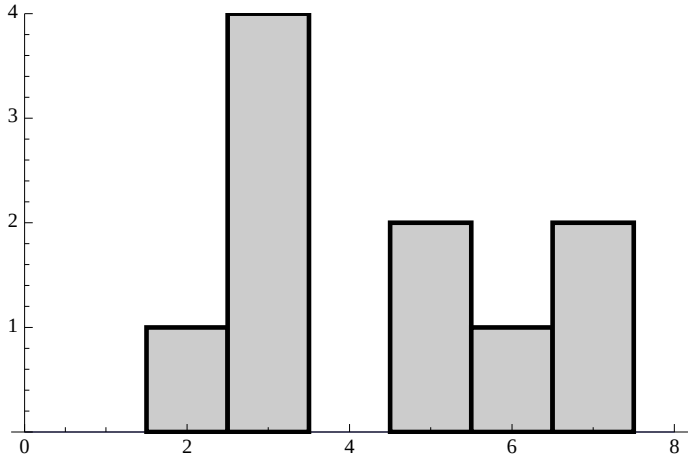
a. Comparative but not an experiment.
b. A comparative experiment but not randomized.
c. A comparative randomized experiment.
d. A matched pairs experiment.
e. An experiment but not comparative.

f. A waste of my time.

MC11. A researcher believes that spiral-horned snorkacks have longer horns than crumple-horned snorkacks. He selects a simple random sample of 5 snorkacks of each kind, and measures their horn lengths. Let μ_1 and μ_2 be the true mean horn lengths of spiral-horned snorkacks and crumple-horned snorkacks, and let $\overline{x}_1$ and $\overline{x}_2$ be the mean horn lengths of the two samples. The correct alternate hypothesis is:

a. H_a: $\mu_1 > \mu_2$.
b. H_a: $\mu_1 = \mu_2$.
c. H_a: $\mu_1 \neq \mu_2$.
d. H_a: $\overline{x}_1 > \overline{x}_2$.
e. H_a: $\overline{x}_1 = \overline{x}_2$.
f. H_a: $\overline{x}_1 \neq \overline{x}_2$.
g. No hypotheses are appropriate, because of a poor choice of sampling method.
h. None of the above.

MC12. The following histogram is made from integer data.



What is the median of the data?

a. 2
b. 3
c. 4
d. 5
e. 6
f. Impossible to tell from the information given.
g. None of the above.

MC13. A deranged official in the UO Registrar's Office wants to find ways in which UO students differ from Oregon State students. He chooses simple random samples of students from each university, and uses the two sample t procedure to test for differences in 20 characteristics (weight, blood pressure, mother's highest educational level, etc.), each at the significance level $\alpha = 0.05$. Just one of these tests, on depression as measured by the Beck Depression Inventory, rejects the null hypothesis at this significance level. Which of the following is it safe to conclude?

a. There is strong evidence that UO and Oregon State students have different mean levels of depression, although the difference may be small and of little practical significance.
b. The significance level $\alpha = 0.05$ greatly overstates the actual significance of the test for different mean levels of depression. This could easily have happened by chance.

c. It is very unlikely that the depression levels of the two samples would have been as different as they actually were if UO and Oregon State students had the same mean level of depression.
d. The Civil War football game just occurred, and one team lost.
e. It is very unlikely that one of the tests would have reached this significance level if UO and Oregon State students were on average the same in all 20 characteristics.
f. Nothing, because of a poor choice of sampling method.

MC14. A moderately small set of data (between 30 and 40 items) is roughly normally distributed, except for one extreme outlier at the low end. The statistician analyzing these data has good reason to believe this outlier resulted from a measurement error, and therefore discards it. How does the new first quartile (of the data without this outlier) compare with the first quartile of the original data?

a. The new first quartile is significantly smaller than the old first quartile.
b. The new first quartile is about the same as the old first quartile.
c. The new first quartile must be exactly the same as the old first quartile.
d. The new first quartile is significantly larger than the old first quartile.
e. Any of the above can happen.
f. None of the above.

MC15. Let μ be the mean score of students at Central Community College (CCC) on the Survey of Study Habits and Attitudes (SSHA), a questionnaire on which scores range from 0 to 200. The test is administered to a simple random sample of CCC students, and the following 95% confidence interval for the mean μ is computed: $87 \leq \mu \leq 106$. Which of the following statements gives a valid interpretation of this interval?

a. 95% of the sample of CCC students would score between 87 and 106 on the SSHA.
b. 95% of all CCC students would score between 87 and 106 on the SSHA.
c. If the procedure were repeated many times, approximately 95% of the resulting confidence intervals would contain the mean SSHA score of all CCC students.
d. If the procedure were repeated many times, approximately 95% of the resulting confidence intervals would contain the mean SSHA score of the sample of CCC students.
e. The sample contained at least 95% of all CCC students.
f. The mean SSHA score of all CCC students is 95.
g. Nothing, because of a poor choice of sampling method.

MC16. A sociologist wants to compare the mean math SAT scores of seniors in two high schools. He chooses a simple random sample of 40 seniors who took the SAT at one school, and a simple random sample of 60 seniors who took the SAT at the other school, obtains their math SAT scores, and wants to use the two sample t procedure to test the hypothesis that mean math SAT scores of seniors in the two high schools differ. If he uses the conservative choice of t distribution, how many degrees of freedom will it have?

a. 60
b. 59
c. 50
d. 49
e. 40
f. 39
g. The procedure should not be used, because the samples were not chosen properly.
h. None of the above.

MC17. The Megalopolis Unified School District has 100 elementary schools. The mean of their enrollments is 400, the median is 350, the standard deviation of their enrollments is 150, and

the range of their enrollments is 800. The total elementary school enrollment in the Megalopolis Unified School District is:

a. 15,000
b. 35,000
c. 40,000
d. 80,000
e. Impossible to tell from the information given.
f. None of the above.

MC18. A researcher wants to study the relationship between size and food consumption in southwestern speckled rattlesnakes. He chooses a valley which looks like good rattlesnake habitat and collects for study all the southwestern speckled rattlesnakes he can find in this valley. The snakes he collected form:

a. A population.
b. A voluntary response sample.
c. A simple random sample.
d. A stratified random sample.
e. A convenience sample.
f. A poisonous random sample.

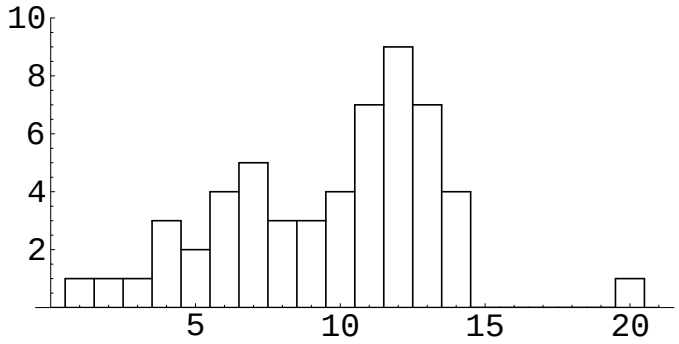
MC19. A professional gambler has a die which is biased in such a way that the probability of getting a 2 when the die is thrown is 1/9. Which of the following statement gives a valid interpretation of this probability?

a. If the die is thrown a very large number of times, the proportion of the throws which give 2 will be very close to 1/6.
b. Every ninth throw of this die will be 2.
c. In any nine consecutive throws of this die, exactly one will be 2.
d. In any nine consecutive throws of this die, at least one will be 2.
e. If the die is thrown a very large number of times, the proportion of the throws which give 2 will be very close to 1/9.
f. The gambler is intending to cheat in a game involving dice.

MC20. As part of an investigation of the mean GPA of students at Oregon State University, its Registrar's office selects 200 students at Oregon State University, and looks up their GPAs in its student database. What is the sample?

a. The courses taken by the 200 selected students.
b. The Registrar's office.
c. All students at Oregon State University.
d. The 200 selected students.
e. The GPAs of the 200 selected students.
f. The GPAs of all Oregon State University students.
g. Impossible to tell from the information given.
h. None of the above.

MC21. Consider the following histogram (made from integer data):



The distribution of the data plotted is:

a. Roughly symmetric with no outliers.
b. Roughly symmetric with one or more outliers.
c. Skewed to the right with no outliers.
e. Skewed to the right with one or more outliers.
f. Skewed to the left with no outliers.
f. Skewed to the left with one or more outliers.

2. Other problems

1. In the following situations, determine which one of the following statistical procedures is appropriate. Fill in each blank with the letter of the correct procedure. (Choices may be used more than once.)

Procedures:

A. one sample z procedure
B. one sample t procedure
C. matched pairs t procedure
D. two sample t procedure
E. one proportion z procedure
F. two proportion z procedure

Situations:

(1) _____ A company claims that its test preparation materials increase the SAT scores of the people who use them by an average of 50 points. We take a simple random sample of 100 students who used the materials and find that their scores increased by an average of 40 points, with a standard deviation of 20 points. Do we believe the company's claim?

(2) _____ A medical researcher has 50 subjects on which to test a new cholesterol lowering drug. For a period of one month, the researcher administers the new drug to some of them and a placebo to others, and measures the change in LDL cholesterol levels. Then he switches treatments for a month, administering the drug to those who got the placebo the first time around and vice versa, and again measures the change in LDL cholesterol levels.

2. We want to estimate a certain true population proportion p with margin of error 0.2 with 99% confidence. How should we choose the sample size in each of the following two situations?

a. We have good reason to believe that the true value of p is between 0.3 and 0.7.

b. We have good reason to believe that the true value of p is close to zero.

3. (Not all the steps have the same point value.) In 1988, the ABC News poll found that 55% of the 1175 people it sampled intended to vote for Bush. The CBS News poll found that 57.5% of the 1870 people it sampled intended to vote for Bush.

Test at the 10% significance level whether there is a statistically significant difference between the outcomes of the two polls. Assume that the samples are simple random samples. Answer using the following steps.

(1) State which procedure you will use (for example, one sample t or z procedure, or some other appropriate procedure from this course).
(2) Check that you can safely use the procedure in this case.
(3) State the hypotheses you will test.
(4) Calculate the test statistic.
(5) Give a P-value (or give two values between which P lies), and illustrate your answer with a graph.
(6) State the appropriate conclusion in ordinary language appropriate to the situation described in the problem.

4. Based on a simple random sample of 250 credit card holders, a department store found that 185 card holders incurred a monthly interest charge on an unpaid balance.

a. Find a 90% confidence interval for the proportion of credit card holders who incur a monthly interest charge. Be sure to check that the procedure is safe to use.

b. How large should the sample size be if we want a 98% confidence interval with a margin of error of 0.03 (or less)?

5. A manufacturer receives parts from two suppliers. A simple random sample of 400 parts from supplier 1 has 20 defective parts. A simple random sample of 100 parts from supplier 2 has 10 defective parts.

a. Determine a 90% confidence interval for $p_1 - p_2$. Be sure to check that you can safely use the procedure you choose.

b. The foreman of the plant claims that there is a difference between the fraction of defective parts produced by the two suppliers. State the hypotheses and check that you can safely use the z procedure. Find the test statistic. What is the P-value? Can the foreman of the plant conclude that there is a difference between the fraction of defective parts produced by the two suppliers at the $\alpha = 0.05$ level of significance?

6. A wizard believes that the sex ratio among Nigerian green dragons is skewed: females outnumber males. He examines 200 Nigerian green dragons, and finds that 112 of them are are female. Assume that the total population of Nigerian green dragons is much more than 200, and that the sample can be treated as a simple random sample. Test the wizard's belief at the 5% significance level, using the following steps.

(1) State which procedure you will use (for example, one sample t or z procedure, or some other appropriate procedure from this course).
(2) State the hypotheses you will test.
(3) Check that you can safely use the procedure in this case.
(4) Calculate the test statistic.
(5) Give a P-value (or give two values between which P lies), and illustrate your answer with a graph.
(6) State the appropriate conclusion in ordinary language appropriate to the situation described in the problem.

7. Repeated measurements of the concentration of potassium ions in a solution with a particular scientific instrument vary normally with standard deviation 1.8 milligrams/liter (mg/l) and with

mean equal to the true concentration. How many measurements are necessary to produce a 99% confidence interval for the true concentration with margin of error 2 mg/l? (If relevant, your work must show the appropriate critical value.)

8. You want to determine whether a particular antidepressant medication can reduce the severity of headaches in people suffering from chronic headaches. You have 200 test subjects available who suffer from chronic headaches. Use a diagram to outline in detail the design of a randomized comparative double blind experiment. Include information about the treatment groups and the response variable. Be sure that one can tell from your description that your experiment has all the characteristics expected of such experiments.

9. Professor Greenbottle wants to estimate the mean IQ of Martian school children. He chooses a simple random sample of 11 Martian school children, and administers an IQ test to each of them. The results are:

93 125 78 132 75 87 82 87 138 149 127

Is it appropriate and safe to use the one sample t procedure on these data to find a 95% confidence interval for the mean IQ of Martian school children? Why or why not?

(Do **not** carry out the procedure, even if it is appropriate.)

10. We are told that the mean diameter of borogove eggs is 4.2 cm. Alice believes this number is larger than the true mean diameter. She collects five borogove eggs, which we treat as a simple random sample of all borogove eggs. Their diameters in cm are:

3.8 3.6 4.2 3.3 4.0

These data contain no outliers and are not obviously nonnormal. Test Alice's belief at significance level $\alpha = 0.05$, using the following steps.

(1) State the hypotheses you will test.
(2) State the test you will use.
(3) Calculate the test statistic.
(4) Give a P-value (or give two values between which P lies), and illustrate your answer with a graph. (If relevant, you must show the number of degrees of freedom used, even if you use a calculator to find the P-value.)
(5) Draw and state the appropriate conclusion, expressing it in words appropriate for the context of the problem.

11. The table below gives the probability that a roll of a particular loaded die gives each outcome.

Outcome	1	2	3	4	5	6
Probability	0.4	0.1	0.1	0.05	?	0.05

Remember that you must **show some work** (even if you can do the problem in your head).

a. Find the probability of rolling a 5.

b. Find the probability of rolling a 1 or a 4.

12. (Work need not be shown.) The five number summary of the final exam scores in a European history class is:

2 71 120 156 199.

The questions below are about the original data, which consists of one number for each of the 37 students in the class. For each of the following statements, circle "A" if the statement must be true, circle "S" if the statement might or might not be true, and circle "N" if the statement cannot be true. In other words, under the stated circumstances the statement is **A**lways true, **S**ometimes true, or **N**ever true.

A S N (a) At least one student received a score of 120.

A S N (b) The score 2 is an outlier.

A S N (c) The mean score is 120.

A S N (d) About 9 students had scores of 71 or worse.

A S N (e) The standard deviation of the scores is 36.

A S N (f) If the instructor divides all the scores by 2 (to make the point total equal to 100), the new third quartile will be 78.

13. The masses of Nigerian spotted dragons are approximately normally distributed with mean 3.41 tons and standard deviation 0.62 tons.

In each of the following parts, show your work, and draw a picture of the appropriate normal curve with the relevant points on the horizontal axis clearly marked and with the appropriate area shaded and clearly identified. The curve must look reasonable.

a. Approximately what percentage of Nigerian spotted dragons weigh between 2.79 and 4.65 tons?

b. Approximately 15% of the Nigerian spotted dragons weigh more than _____ tons.

15. For a biology project, a student measures the front tooth length (in centimeters) and the mass (in kilograms) of each of 11 alligators. What units of measurement do each of the following have? (Write "none" if appropriate.)

a. The mean of the masses.
b. The standard deviation of the tooth lengths.
c. The z-score of the mass of the last alligator.
d. The correlation between mass and tooth length.

16. Repeated weighings of an object with a particular balance give results which vary normally with standard deviation 1.8 milligrams (mg) and with mean equal to the true mass.

a. The true mass of one object is 60.1 mg. If the object is weighed 4 times with this balance, what is the probability that the mean of the measurements is at least 60 mg? Illustrate your answer with a graph in which the appropriate areas are labelled.

b. For another object, nine weighings with this balance gave $\overline{x} = 55$ mg. Construct a 90% confidence interval for the true mass of this object.

17. Male Martian teenagers have weights (in pounds) which are normally distributed with mean 516 and standard deviation 114, and female Martian teenagers have weights (in pounds) which are normally distributed with mean 412 and standard deviation 100. A particular male Martian teenager weighs 680 pounds, and a particular female Martian teenager weighs 560 pounds. Find the standardized weights for both Martians. Which one is more overweight?